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Qualitative reasoning over time: history and current prospects

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Abstract

This paper provides a historical summary of the motivations which have led several research communities to contemplate qualitative techniques. Qualitative reasoning satisfies various problem solving needs in high level decision tasks, embodied in a set of tools which allow deep knowledge to be put in compatible form with software requirements while still remaining realistic. An overview of these mathematical formalisms is presented; qualitative simulation is introduced as one of the most significant outcomes. Finally, some current research issues concerning temporal aspects of qualitative reasoning are discussed.

1 Introduction

Artificial intelligence techniques appear today as a major alternative to conventional engineering methods when problem complexity demands use of expert domain knowledge. "Shallow" knowledge-based expert systems have provided significant advances in this field. This type of system is principally based on a representation of experts' knowledge. This endows them with appreciable advantages, such as flexibility to represent many types of phenomena, but at the same time results in serious limitations. Three essential reasons can be identified:

- shallow knowledge only encodes *a priori* known situations;
- shallow knowledge does not capture the causal nature of phenomena;
- explanation is typically limited to a mere sequence of rules

These limitations can be overcome if some "deep" knowledge exists in the application domain. While shallow knowledge encodes an expert's understanding of a domain, deep knowledge encodes a model of its laws or first principles.

Today, expert systems increasingly combine shallow and deep representations so as to obtain the advantages of both. Routine problem solving in well-known situations can take advantage of reasoning shortcuts, while deep knowledge endows the system with the flexibility to *generate* solutions from first principles in unfamiliar ones.

Engineering applications best illustrate the potential for exploiting underlying physical laws, and often also permit knowledge of functional relationships and structural information to be used.

If deep models are to be used by a problem solver, they need to be realistic while still being practical given software and computational resources. Engineers can often derive very significant conclusions from qualitative considerations, and this led to the development of *qualitative reasoning*, which has wider applicability than just modelling physical systems. Qualitative reasoning has been a very active research area over the past decade (*Artificial Intelligence*, 1984). However, the idea of a qualitative representation of the world is not new; long before it reemerged in AI, the word "qualitative" had been used in various communities interested in system theory, including economics (Lancaster, 1962) and automatic control (Travé & Kaszkurewicz, 1986).

These communities were mainly motivated, on the one hand, by the lack of quantitative data for building conventional numerical models, and on the other hand by the wish to distinguish between conclusions derived from specific numerical patterns in the model, and from general structural features. Qualitative techniques were disregarded for several years before being applied in AI, through qualitative physics. The AI community's interest in the qualitative approach has stimulated other communities to take an interest in these techniques, and qualitative reasoning techniques have themselves greatly benefited from AI techniques.

Qualitative reasoning nowadays has overlapping motivations; it tries to address both the need to deal with systems which are not easy to quantify, and the desire to emulate the human ability to reason at a qualitative level. For this purpose, qualitative reasoning combines established techniques (qualitative calculus etc.) with novel AI-based ones (constraint propagation, logic programming etc.). It provides general modelling formalisms which may be used in numerous fields as well as qualitative simulation algorithms, assisting future trends to be predicted from a qualitative model. Undoubtedly, this is a key capability for second generation expert systems.

2 Historical overview

In 1977, naïve physics was developed around the idea of providing computers with models of the world which would agree with human perception. Indeed, it was thought that such models might capture the whole of human commonsense knowledge (Hayes, 1977). During the same year, the "Artificial Engineer" project began at MIT. The goal of the project was to develop commonsense knowledge to provide a model of the physical world based on engineers' knowledge and expertise. This was the starting point for qualitative physics, which then developed as a research area in its own right, and from there to a more general study of qualitative reasoning. Looking back, however, one cannot fail to notice that the idea of a qualitative description of the world has been used frequently.

As early as the 1960s, a qualitative *catastrophe theory* was proposed by the mathematician René Thom (1977). Thom noticed that humans think quantitatively, but rarely go into arithmetic; results are always qualitative, and this is much more appropriate for every-day actions than real numbers.

At about the same time, economists were actively engaged in developing techniques for qualitative system-analysis. Economic theories generally deal with very complex systems for which complete quantitative models are neither known nor likely to be found. Theorists believed that many economic propositions could, however, be expressed in a qualitative form obtained solely from a knowledge of the signs of relevant structural parameters, and that they could then be used for predicting variational trends. In the case of numerical analysis with a pure numeric model, the results do not allow any distinction between conclusions stemming from the numerical details of the model and those coming from its structural invariants. Qualitative analysis articulates this distinction.

A matricial qualitative calculus based on *sign algebra* was developed, and many interesting approaches to theoretical problems like qualitative determinacy and qualitative solvability (Maybee, 1981) were developed. But the heart of qualitative economics was primarily concerned with problems in comparative statics (Ritschard, 1983). Comparative statics relies on comparing the equilibrium configurations of two economic systems which are identical in all but a small set of clearly defined parameters. The method is based on the suggestion that those entities which do not differ from one system to the other should not have to be quantitatively specified. Comparative statics is often used to examine the effects on a given economic system of a change in one or more of the parameters. In simple static terms, the prediction can be made qualitatively. Thus the problem consists in determing the direction of change of the endogeneous variables of the economic model, given the direction of change of the exogeneous variables and the signs of the partial derivatives of the corresponding system of equations. In such problems, qualitative determinacy and qualitative solvability issues are obviously of major importance.

The automatic control community also showed an interest in qualitative studies. Their objective was to derive system properties from their qualitative characteristics. In particular, they aimed to provide criteria which would guarantee the properties of dynamical linear systems from qualitative models (Travé & Kaszkurewicz, 1986). Studies mainly focused on linear systems which can be represented in the state space formalism as

$$dx(t)/dt = Ax(t) + Bu(t)$$
$$y(t) = Cx(t)$$

where $x(t)$, $u(t)$ and $y(t)$ are the state, input and output vectors, respectively and A, B, C the matrices of appropriate dimension.

The qualitative model is based solely on the knowledge of interconnections within the system and their direction. The form of the state space model is preserved in all but matrices A, B and C which become signed matrices, i.e., their parameters are either zero, plus, or minus.

Such a qualitative model represents a whole class of real systems. Hence, qualitative properties have a great robustness advantage, since they are shared by a whole class of real models consistent with the qualitative model, i.e., for which real matrices (A, B, C) are consistent with the sign patterns of qualitative matrices. This is particularly useful when dealing with large, complex systems for which it is impossible to guarantee accurate data. Also, in the case of non-linear control systems which use the same linearized system structure for every operating point, qualitative analysis allows us to state the properties of every operating point model in one step.

The criteria allowing these properties to be expressed are based on the same sign-algebra-based matricial qualitative calculus as used by economists. Indeed, studies in economics and automatic control can be viewed as belonging to a general qualitative system theory. The results are generic since they are obtained from general models of differential equations. Therefore, they are potentially applicable to many other areas as long as systems specific to a domain can be represented in the appropriate mathematical form (Travé-Massuyès, 1989).

In spite of their power and generality, use of qualitative techniques was limited until they were adopted in AI. Then several scientific communities interested in AI's potential became interested and began to use them for their own purposes. The AI community did not always approve of this, as qualitative techniques were often used for purposes other than those initially intended in AI. When the word "qualitative" reemerged in AI, motivation was supported by the desire to *build computer systems* with the ability to reason about physical systems in the same way as humans—or at least engineers—do, which implied reproducing mental models used by humans to reason about such systems. An *intermediate* objective was therefore to build models from which a behavioural description of the systems could be derived in terms of its qualitative features (relevant value changes, direction of change switches, order of magnitude information, etc.), bearing in mind that the *final goal* was that the computer itself would use them to initiate its self-contained reasoning. Nevertheless, qualitative techniques were gradually adopted by engineers to solve their problems rather than to have problems solved by the computer. As a result, qualitative techniques were progressively studied for their own sake, even in AI.

In Europe at least, although the original AI goal is always implicit, much of the current qualitative reasoning literature is oriented towards solving the engineering problems (Preprints, 1991). In France, this situation is clearly illustrated by a national project *Qualitative Modeling and Decision*, which gathers together *all* French research groups involved in projects which include qualitative reasoning, from universities to the private sector (MQ&D Project, 1991).

However, there is no doubt that qualitative reasoning techniques have greatly benefited from being associated with AI techniques, and from being highlighted by new AI motivations. This has been fruitful for all communities.

The two major contributions of AI have been:

- the introduction of a *dynamic dimension* to qualitative systems analysis. Previous studies by economists and automatic control researchers dealt with static problems, and never addressed the issue of predicting the *dynamic behaviour* of systems (running qualitative simulations);
- recognition of the need for *explanation*; the objective is not only to provide results but to be able to account for them as well.

These two aspects are central to the new direction taken by research in qualitative reasoning.

3 Current prospects for qualitative reasoning

3.1 Needs

Information processing for management and control of systems and organizations can be decomposed into two different layers as shown in Figure 1.

The bottom level is concerned with *control tasks* which can be easily automated. Indeed, there is generally a one-to-one correspondence between the system state and the action that must be performed to achieve a given objective. Problems addressed here are generally specific and environmental limits well-defined. Moreover, the tasks are very often repetitive. Hence, the available information often permits accurate numeric models to be constructed and decisions may generally be taken with algorithmic methods. This layer usually operates in real-time, and the actions can be viewed as *reflex actions*.

On the other hand, the top level includes *decision making tasks*. Problems are generally embedded in a complex environment so that several alternative actions exist; the "good" action is not necessarily the same for two *apparently* identical states. Here the knowledge used by experts becomes increasingly disparate. It still includes quantitative data, but as soon as decision-making tasks are involved this data can no longer be directly used (Sage, 1990; Singh, 1991; Travé-Massuyès, 1991). Decision making generally requires taking into account the state of the system (quantitative and qualitative), as well as the surrounding environment. Information is always incomplete and inaccurate. Such decisions are governed to a great extent by "expert intuition".

Intuition is difficult to capture as it can be viewed as mixed highly aggregated knowledge and meta-knowledge. Expert systems are typically designed to operate at this level. Should models be used by the problem solver, they need to be realistic enough while still being compatible with computational resources. *But realistic numeric models cannot often be derived from such miscellaneous data. Even if they could, they would not be appropriate to decision tasks.* Data must be processed at the right level of abstraction which must be consistent with the data supporting expert

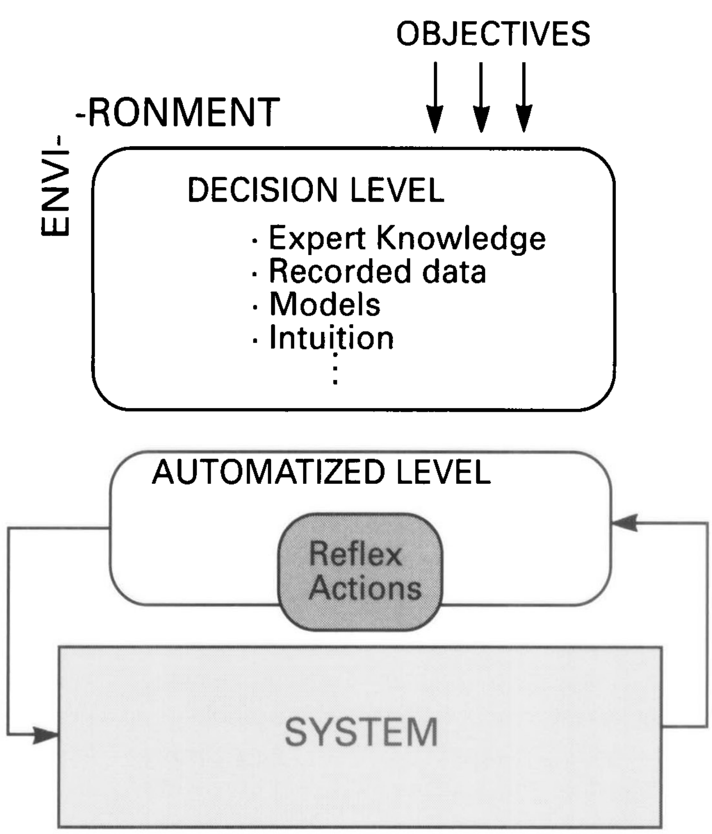

Figure 1 Management and control of systems and organizations

reasoning, i.e., highly aggregated data expressing *qualitative features* and *cause-to-effect relationships*. Furthermore, rather than aiming at providing a detailed plan or numerical solution, it is more appropriate that models provide help in tendency analysis, and in early detection of the system's current operating state. It might also be desirable to have an *explanation* of the results.

As will be brought out in the next section, these needs can be met by qualitative modelling and qualitative simulation.

3.2 Objectives of qualitative reasoning

The power of qualitative reasoning can be related to the fact that it gathers all the aspects which successively motivated the different research communities. It addresses the issue of dealing with systems which are not easy to quantify, and that of capturing the human ability to reason at a qualitative level. Established and novel AI-based techniques are combined to provide much greater effectiveness.

Qualitative economic and automatic control models were mainly obtained from the theoretical laws underlying the application domain, or by taking an abstraction of known quantitative models. Hence, they entirely relied on deep knowledge, and remained rather limited in their ability to describe realistic situations. The challenge to qualitative modelling is to encode shallow expert knowledge and deep knowledge in the same formalism.

Expert-knowledge is generally expressed in terms of *approximate data* and relations which give information about qualitative properties rather than numerical values. Very often, experts know that some threshold is included in a numerical interval, or that a variable value has some numerical range. They may also indicate the related tendencies of variables and their orders of magnitude. On the other hand, when deep knowledge is available, in terms of theoretical laws, it is generally impossible to provide accurate values for all the parameters involved because of *imprecise data*. It follows that one of the major issues for qualitative modelling is *approximate/inaccurate data representation and manipulation*.

In this respect, it may be worth distinguishing the words *qualitative and fuzzy*; the terms cover different though complementary underlying concepts. Where a qualitative representation handles *inaccuracy* a fuzzy representation allows for some *refinement related to preferences between possible values*. In fuzzy set theory, preference is formally captured by membership functions. This is why both formalisms can be associated when the problem involves inaccurate data (Dubois & Prade, 1988, 1989; Shen & Leitch, 1990).

Another issue which has only been formulated clearly quite recently, is the need to handle inaccurate along with precise data, i.e., using both qualitative and quantitative knowledge. The pioneering investigations were carried out within a pure qualitative framework (*Artificial Intelligence*, 1984), but the last ten years' experience has shown that:

- a qualitative description is often insufficient for drawing relevant conclusions due to the inaccuracy propagation cone which increases widely;
- a fully qualitative description may not take advantage of all available knowledge. Available knowledge is often a mix of precise and inaccurate data. Also, it frequently happens that one wishes to describe the effects of variations of some specific parameter(s) while keeping the other one(s) fixed. Pure qualitative formalisms do not allow for fixed real values to be set. As all parameters are allocated a qualitative value, significant effects may be lost while simultaneously propagating the whole inaccuracy.

As pointed out in a moment, handling qualitative and quantitative knowledge is a stringent requirement that calls for combined numeric-symbolic formalisms. Active research is being conducted in this area, and some results have already been reported (Dague, 1988; Kuipers & Berleant, 1988).

Also worthy of note is the slightly different issue of integrating qualitative models with quantitative ones. This entails interesting problems like selecting the most appropriate model for the level of abstraction required, and designing qualitative/quantitative interfaces. These issues are being particularly investigated in Europe.

Thus far, the type of expert knowledge mentioned can be viewed as a substitute for unavailable deep knowledge. However, another type of knowledge not captured by mathematical equations is the underlying *causality* of the system. Experts often refer to the causal relations between phenomena, and it is generally possible to build a causal network of the processes involved. Qualitative models must also therefore be able to reflect causal knowledge. This is underlined by the other claim of qualitative reasoning to be able to provide not only analysis tools, but also tools for *explaining the results* of the analysis.

However, causality may be defined at different levels. Among the formalisms presented in the next section, a distinction will be made between those encoding *non-oriented relationships* and others encoding *oriented relationships*. Two types of qualitative models follow, depending on the formalism which is being used:

- non-causal models which only express algebraic constraints between the values of the variables, like conventional equations;
- causal models which encode the underlying causality within the system in addition to algebraic relations.

In causal models, causality is *a priori* knowledge furnished by experts. In this respect, there is much debate going on between those who favour non-causal models, thinking that some features of the system may be ignored by defining causality only as humans presume it to be and those who consider that causal knowledge is worth using if it is available.

The problem can be expressed in a different way: causal models are a good alternative when deep knowledge in equational form is not available. Indeed, most of the mental models used by experts embody a causality network so that it is generally possible to work out a causal model from expert-knowledge. On the other hand, causal models are not generally invertible, i.e., knowledge cannot be propagated in both directions.

Insofar as the explanation issue is concerned, humans are apparently unable to do without causality. Explanation is facilitated when using a causal model, because the causal graph can be retraced to establish reasoning paths. Explanation is not so easy with a non-causal model, as is clearly shown by the number of studies on this topic (de Kleer & Brown, 1984; Vescovi, 1991; Iwasaki, 1988). Starting from a functional model in terms of non-directed algebraic equations (confluences, *QDEs*, etc.), the problem is to recover causal links between variables. For instance in de Kleer & Brown (1984), *mythical causality* is supposed to recover the pathway followed by a disturbance propagated through the system. On the other hand, *causal ordering* techniques proposed in Iwasaki (1988) are based on finding out independent subsystems of equations within the functional model.

Now the question is to know whether the causality recovered by such techniques is always consistent with causality as understood by system experts. This should be so if it is to be in agreement with dynamical systems theory. In this theory, the usual assumption is that dynamical systems are non-anticipatory (causal), which understands the existence of cause-effect relationships as implying that the cause and the effect can be *unambiguously* determined.

However, causality as used in qualitative reasoning is not always consistent with this view. Indeed, what is important in dynamical systems theory is that causality is closely related to running time. It is always assumed that there is a time lag between the appearance of cause and effect, that is, *cause and effect can be chronologically ordered*. Surprisingly, in qualitative reasoning there have been many publications on causal ordering from static models of the system. In this case, it is unsurprising that the ordering may be ambiguous, or even contradictory to common sense. It seems clear to the author that causality is related to dynamics, and that it cannot be recovered without using a dynamical model. Some recent results by Iwasaki (1988) can be mentioned in this connection.

However, the question of whether recovered causality corresponds to causality as presumed by experts still remains. It is not easy to answer, because when a mathematical model (even qualitative) based on theoretical laws is available, we tend to trust it, and not think it worth questioning the expert. The expert himself often reasons from the equations. Conversely, when we build a causal model, it is generally to compensate for the absence of deeper knowledge. As a result, no significant comparison has been made so far, and this remains a direction for research and experience.

4 Qualitative formalisms

This section presents mathematical formalisms used in qualitative reasoning which allow approximate data or relationships to be represented. Qualitative concepts can be associated with physical entities, or relationships between these entities, by means of symbols. The semantics of these symbols express qualitative properties currently used for qualifying numerical entities.

4.1 Representation of approximate data; qualitative algebras

The problem can be formulated as follows: how to leave conventional representation of real numbers and pass on to more "aggregated" ones. A physical entity must no longer be qualified by numerical values but by symbols which encode qualitative properties, e.g., small, medium, large. On the other hand, symbolic mathematical structures must still allow symbolic manipulations governed by *propagation rules* or *qualitative calculus laws*. The structures presented in this section preserve consistency with conventional manipulations of real numbers, which is particularly important if one wants to deal simultaneously with qualitative and quantitative data.

Qualitative representation is based on partitions of the real line (for instance, see Figure 2). Thus, a symbolic variable corresponds to a subset of $\mathfrak{R}$. As these variables represent physical numeric entities, algebraic manipulations like sum or product must remain consistent with sum and product as defined in $\mathfrak{R}$. Qualitative sum and product, $\oplus$ and $\otimes$, are therefore chosen as *qualitative functions* associated with corresponding real functions (Missier et al., 1989).

Most studies rely on the concept of *qualitative equality* whose semantics is "*possibility of being equal*". In other words, two symbolic objects are qualitatively equal if the subsets of reals that they represent have a non-empty intersection. An axiomatization of qualitative equality consistent with this semantics was provided in (Travé-Massuyès & Piera, 1989). A major difficulty in qualitative calculus lies in that *qualitative equality is not transitive*.

Note that interval algebra is a limit case of qualitative algebras for which the partition is infinite and the boundaries are themselves real numbers. In fact, qualitative algebras can be ranged in a continuum going from the least informative partition (sign algebra) to the most informative one (interval algebra, which can also represent real numbers). Interval algebra has been used in several applications as it is very suitable for dealing with inaccurate data when inaccuracy is expressed in terms of values belonging to some numeric interval. Furthermore, interval algebra is appropriated for dealing with mixed data, accurate and inaccurate. As it has been deeply investigated for a long time, it is not developed further here. Rather the focus is on newer structures which were developed specifically for qualitative reasoning.

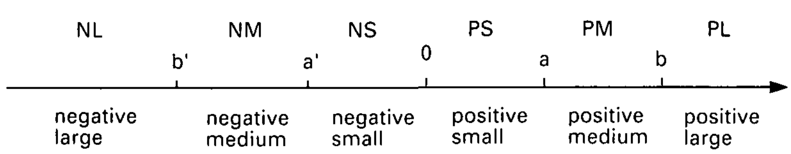

Figure 2 A special partition of the real line

4.1.1 Sign algebra

This algebra is based on the roughest partition of the real line: negative numbers, positive numbers and zero. The set of symbols is then $S = \{+,-,0,?\}$, where *?* stands for undetermined sign. *?* is necessary to obtain closeness of sum and product operators. Operators are defined in a trivial way, e.g., $+ \oplus + = +$, $+ \oplus - = ?$, etc.

Although the sign partition roughness induces some limitations, which will be discussed later on, it also offers several significant advantages. The major advantage concerns the interesting mathematical properties of sign algebra:

- qualitative equality is transitive if the middle element is different from *?*;
- sum and product are associative.

This makes qualitative calculus rather feasible. In particular, a qualitative resolution rule has been proved (Dormoy & Raiman, 1988). Sign algebra is very well-suited to tendency modelling (increasing, decreasing or steady variables) when employing the signs of the variable changes. Hence, sign algebra is one of the most often used tools in qualitative reasoning.

4.1.2 Order of magnitude qualitative algebras

Limitations of sign algebra were rapidly pointed out. A major difficulty lies in that the result of an operation is often related to the order of magnitude of the parameters. Sign knowledge by itself often leads to undeterminations, e.g., $+ \oplus - = ?$. More sophisticated structures based on finer real line partitions were then proposed to capture orders of magnitude (Travé-Massuyès & Piera, 1989). These algebras take into account some quantitative information by choosing *numerical boundaries* which specify a partition of $\Re$. It is therefore impossible to always remain at a symbolic level, and some steps require explicit reference to real numbers.

This produces the fundamental difference relative to sign algebra, since a given set of symbols corresponds here to an infinity of partitions of $\Re$ (related to the choice of numerical boundaries a, b, a', b' for the Figure 2 example). Consequently, symbolic tables for $\oplus$ and $\otimes$ which preserve consistency with real numbers are not unique. However, their number is not infinite. Indeed, there exists a partition of the boundary space into equivalence classes, each one corresponding to a different symbolic table.

This kind of study has been developed for the partition given in Figure 2 (Piera et al., 1991), where boundary space is the connex domain of $\Re^4$ given by $D = \{(a,b,a',b') \in \Re^4, b' < a' < 0 < a < b\}$. For $\oplus$, it has been shown that 201 tables are possible in the general case. However, it is worth noticing that these 201 tables come down to 2 qualitatively different[1] ones only. Note also that in the most commonly used symmetrical case ($a' = -a$, $b' = -b$) there are only 3 possible tables, which also come down to 2 qualitatively different tables. Robustness and precision of these tables have been studied.

Of course, the above point increases the difficulty of problem formulation. First, the most appropriate partition must be determined for each variable and brought back to a given unique partition by a normalization process. The symbolic tables of $\oplus$ and $\otimes$, which will be used for the calculus, are determined for the latter partition. In return, it is very easy to handle quantitative information.

These mathematical structures do not have the same interesting properties as sign algebra so that qualitative calculus is more difficult (Missier et al., 1989; Missier, 1991), in particular solving equation sets becomes very complex. Besides non transitivity of $\approx$, two significant problems are encountered: *lack of associativity* of operators $\oplus$ and $\otimes$, and *lack of distributivity*. In other words, the same arithmetic expression yields different results depending on processing sequence. It is reassuring that the results are always qualitatively equal, but this means that "minimality" of the

[1]Qualitatively different means that there exists at least a pair of values for which the results are not qualitatively equal in both tables.

solution is not guaranteed. In a simulation framework, this is one of the causes of spurious[1] behaviours. Nevertheless, particular partitions can be proposed to overcome some of these problems (Travé-Massuyès et al., 1990). Preliminary discussions of the advantages and disadvantages of fuzzy partitions can be found in Dubois & Prade (1988).

4.2 Representation of qualitative relationships

4.2.1 Confluences

Confluences are multivariable linear equations whose parameters take qualitative values. The term "confluence" was introduced by De Kleer & Brown (1984). In their approach, a qualitative model consists of a set of confluences whose parameters take values in $\{+, 0, -\}$. Confluences express the constraints linking *variations* of significant variables in the neighbourhood of an equilibrium point. They may be derived by differentiating physical equations of equilibrium, for instance. Within this approach, solutions must be interpreted as infinitesimal variations around this point.

This approach is closely related to economists' work, which already addressed the problem of solving confluences by means of sign algebra (Maybee, 1981; Ritschard, 1983). This issue was also deeply investigated in qualitative automatic control (Travé & Kaszkurewicz, 1986). However, new interesting results were derived by AI researchers (see in particular Dormoy & Raiman, 1988, and Travé & Dormoy, 1989). The problem was also approached by means of order of magnitude algebras (Missier, 1991; Travé-Massuyès et al., 1990).

4.2.2 Relative order of magnitude models

This approach allows qualitative comparison of entities without referring to any absolute scale. Unlike information provided by qualitative algebras, the information for a given variable is always expressed relative to other considered variables.

The originator of these models is the formal system FOG presented in Raiman (1986). FOG is based on three operators expressing the relations *negligible in relation to, close to, has the same sign and order of magnitude as*. FOG includes one axiom and 30 inference rules which allow propagation of initial knowledge.

Operators are used to express relationships between variables, thus providing a static model of the system, at some operating point.

Other formalisms, derived from FOG, were later proposed to provide solutions to some unsolved problems (Dubois & Prade, 1989; Dague, 1988; Mavrovouniotis & Stephanopoulos, 1987). In particular, major issues were to make the system accept purely numerical values, and to provide a satisfactory answer to the problem of non real transitivity of operators.

4.2.3 Qualitative transfer functions and propagation functions

This formalism is inspired from automatic control theory, in which transfer functions are used to relate linear systems output variables to input variable changes. To a temporal function as input another temporal function is associated as output.

In a similar way, qualitative transfer functions (QTF) define a behavioural constraint between input and output variables, and capture the type of constraint as well as experimental data like qualitative values of gain, delay, settling time, etc. They rely on discretizing time in episodes so that the output temporal function is defined *a priori* by a piecewise linear function (Caloud, 1988; Montmain et al., 1990). The *a priori* shape of the function is an approximation of well-known responses of numerical transfer functions to classical inputs (steps and ramps). As the output provided by a QTF is a linear piecewise function, it is suitable as input to another QTF, which allows propagation of a temporal function through a cascade or a network of QTFs.

[1] However, in this case spurious behaviours are a *subset* of those which would be generated with sign algebra (Misssier et al., 1989).

Propagation functions (PF) (Vescovi, 1991) also define a behavioural constraint between system input and output variables. However, the approach is different in the sense that only steps are considered. PFs express the response to step inputs. Input signals (ramp, exponential, etc.) are approximated by a sum of step signals (called *perturbations*) whose magnitude is significant to process experts. Besides, PFs are defined as *fuzzy-valued* continuous functions, providing an example of qualitative/fuzzy formalism. Propagation is similar to the previous case.

4.2.4 Qualitative differential equations (QDE)

Whereas confluences are static equations, QDEs use a derivation qualitative operator. This allows for a deeper interpretation, going beyond infinitesimal variations in the neighbourhood of an equilibrium point. Given a system, the idea is to build up a model from a limited number of qualitative operators which capture the whole range of possible relationships between variables (Kuipers, 1986). In Kuipers' formalism, these operators are

- Derivation $DERIV(x,y)$ for $y = dx/dt$,
- Sum $ADD(x,y,z)$ for $z = x + y$,
- Product $MULT(x,y,z)$ for $z = xy$,
- Increasing or decreasing monotonicity $M+\ (x,y)$, $M-\ (x,y)$ for $y = f(x)$, where f is an increasingly or decreasingly monotonic function.

The model is therefore given by a *qualitative differential equation* which contains several parameterized differential equations (i.e., whose coefficients are defined in an interval of $\Re$, also called *constraints*).

5 A major outcome: qualitative simulation

In current applications, qualitative models are often used as static descriptions to check out whether real observations—measured variable values—are consistent at some given time point. This approach has already provided significant results (De Kleer & Williams, 1987; Struss & Dressler, 1989), in particular for diagnosing electronic circuits. Nevertheless, the potential of qualitative reasoning techniques relies on the possibility of predicting future behaviour of systems; i.e., providing a view of variable trajectories on a given time horizon.

Although they can provide models to be used in the static approach, most of the qualitative formalisms presented in section 4 have been designed to be applied in qualitative simulation algorithms. A system's future behaviour is predicted from a given initial state in terms of equilibrium states, as well as transient states between two equilibriums.

Actually, qualitative simulation rarely predicts one unique behaviour. It generally provides several sequences of states which are all consistent with the initial qualitative model, each reflecting the particular behaviour of some specific numerical model consistent with the qualitative description. Hence, qualitative simulation is *complete*. Unfortunately, it is also incorrect as qualitative reasoning may not be powerful enough to discard all impossible behaviours. Great efforts have been dedicated to identify incorrectness sources and provide methods which allow the discarding of these *spurious behaviours* (Kuipers & Berleant, 1988; Missier & Travé-Massuyès, 1991).

States are given in terms of variables taking such and such qualitative values, and having such and such direction. Hence, qualitative simulation provides a behavioural description which retains qualitative aspects, and removes insignificant distinctions which would not be consistent with the available knowledge.

As qualitative models and simulation are essentially concerned with dynamical *continuous* systems, it is interesting to notice that they can be interpreted as a discrete-event-based description of the system, in which events are generated by continuous variables crossing thresholds. This approach thus establishes a bridge between continuous and discrete systems theories. Relationships between these theories become obvious when one considers continuous systems and high-level control tasks, which are better performed from alarm message information rather than from continuous observations. The transition between a continous system and a discrete-event description is precisely what is performed by qualitative simulation, and this is one of its attractive features. Investigations are going on in this direction, and promising results can be anticipated (Bousson & Travé-Massuyès, 1992; Lunze, 1990).

Another obvious advantage of qualitative simulation is in cases of incomplete and inaccurate knowledge about the system. Indeed, numerical approaches require the parameters to take some specific numerical values which generally do not have real interpretation, but are rather approximated from qualitative data. For the same qualitative data, several numerical samples are necessary to give a complete description, and several simulations must be run. The advantage of qualitative simulation lies in that it provides *all* possible future changes in one step, this being obtained by using a model simply derived from the available data.

Application fields of qualitative simulation are numerous, and in a wide variety of domains (chemistry, aeronautics, ecology, medicine, economy, business, management, etc.). Let us mention, for instance, model-based diagnosis, industrial process supervision, situation assessment and interpretation, computer-assisted design, etc.

Following the taxonomy used for qualitative models, qualitative simulation algorithms can be classified as *causal* and *non-causal*, depending on the formalism they rely on. A major difference between these two classes can be seen in the fact that non-causal simulation is closer to conventional simulation in the sense that, from a set of equations, the algorithm really retrieves a system's behaviour. On the other hand, causal simulation is closer to a declarative approach, since local behaviours must be provided from expert knowledge, and the algorithm itself has only to recover global behaviour using propagation. These two classes of algorithms are briefly presented in the following. Details of the methods can be found in the references.

5.1 Non-causal qualitative simulation

This section basically concerns simulation algorithms proposed, on the one hand, by De Kleer & Brown (1984) and, on the other, by Kuipers (1986).

De Kleer and Brown's approach provides an *envisionment*. The system is represented by a confluence-based model. The set of confluence solutions is first determined by qualitative calculus (see section 4.1). These solutions represent admissible states of the system. These states are then chronologically ordered in a *mythical time* axis by using heuristics derived from the properties of continuously differentiable functions (intermediate value theorem, mean value theorem, etc.). Envisionment takes the form of a diagram in which possible system behaviours appear as sequences of states which the system may go through.

Kuipers' algorithm uses a QDE-based model. Starting from an admissible initial state, the algorithm finds all possible successor states by using a table of authorized transitions. This table relies on the properties of continuously differentiable functions (notice the overlap with the De Kleer and Brown approach). The second step uses the QDE model to filter out states which are inconsistent with some constraint in the QDE. Because of inaccuracy inherent in the model, there are generally several admissible successor states. Hence simulation provides a tree whose branches represent possible behaviours in terms of sequences of states. Numerous improvements have been made to the original algorithm (Kuipers & Berleant, 1988).

5.2 Causal qualitative simulation

The system is represented by a directed graph encoding the underlying causality. Nodes stand for relevant system variables, whereas arcs reflect influences existing between these variables. Knowledge about some variables is propagated through the graph to obtain knowledge about unknown variables. This approach can be used either for static or dynamic problems.

For static problems, the graph propagates non-temporal qualitative values. An example can be found in Guerrin (1990), in which propagation is performed either directly from one origin to one target variable, or by qualitative operators which combine the influences of two origin variables on the target variable.

For dynamic problems, an explicit representation of time is required. Knowledge about variables may be expressed in terms of temporal functions showing episodes of time (see section 4.2). Propagation of these temporal functions is performed by associating either QTFs as in Caloud (1998) and Montmain et al. (1990), or PFs as in Vescovi (1991). Another recent approach based on the concept of *qualitative automaton* will be briefly presented in section 6.

6 A current research concern: time representation

The event-driven approach used in qualitative algorithms is attractive from the point of view of computation time and storage memory. However, as qualitative models and simulation are essentially concerned with dynamic *continuous* systems, time cannot easily be done away with when dynamic motions are retrieved. Very often, situation assessment depends to a great extent on state duration as well as events' dates. But the introduction of the variable "time" greatly increases algorithm complexity, so that the problem is very often avoided.

Temporal aspects are undoubtedly one of the central research issues for qualitative analysis of dynamical systems. In fact, potential qualitative reasoning applications are closely dependent on advances in this direction. Problems with representation and management of time have been discussed for a long time by researchers in the field, but very few applications have been done so far.

QTFs and PFs are the only formalisms allowing explicit time representation. Nevertheless, the variable "time" is approached very differently from conventional simulation. In conventional methods, although time is discretized, it operates as any other variable in computations, by its value. Every "event" can thus be associated with a time point (even if subject to discretization inaccuracy), and temporal correspondence between such and such event and state duration are "discovered" by the simulator.

In QTFs and PFs qualitative simulation algorithms, temporal information is obtained from *initial temporal knowledge* which must be provided to the system as part of the expert knowledge. This knowledge is already very rich, since every QTF or PF must specify *delay, response time* and any other important temporal feature relative to how a perturbation signal propagates from the source to the target variable. New temporal information then results from *combining* these pieces of temporal knowledge.

In non-causal simulation algorithms, time is approached very differently and closer to conventional methods. There is, however, the singularity that, although it is essentially concerned with continuous systems, time comes as in conventional discrete-event simulation systems. Indeed, the result of the simulation is expressed in terms of a sequence of events, showing chronological order. Neither state duration nor time point value information is available. As this may become a problem in many applications, several publications have been devoted to this issue.

In the rest of the section, some recent ideas and investigations are presented for non-causal as well as causal simulation.

6.1 Non-causal algorithms

The case of QDE-based simulation (QSIM algorithm) is considered more precisely. Although numerous improvements have been added to the first version of QSIM, some problems still remain for obtaining and using temporal information. This problem has been approached in Shen & Leitch (1990) and Kuipers & Berleant (1988) using partial numerical information, but no answer was provided at the neighbourhood of critical points.

In the following, the key ideas of a method based on second order Taylor formulae (Missier & Travé-Massuyès, 1991) are presented as an alternative solution to approach time management in QSIM-like algorithms.

In QSIM, time is represented by symbolic values, which correspond to event occurrence time points. Events correspond to some variable reaching or leaving a landmark value (some specific value in its quantity space).

Time is implicitly presented in QDE models by derivative constraints. However, the variable time is not proceeded in the same way as other variables. When regular variables are related through a qualitative constraint (see section 4.2), they have generally associated *corresponding values* which indicate one landmark value in each of their quantity spaces for which the constraint is known to be satisfied. Hence, corresponding values allow us to "stick" variable motions at some "fixed points". In the algorithm, corresponding values are not considered for the variable "time", and this is one reason why the only correspondence between time and events is chronological order. On the other hand, even if a more sophisticated correspondence was restored, it would often be irrelevant as far as it remained symbolic.

To express durations we need to restore a *numerical ratio* between time and other variables, which can be performed with derivative constraints by using *orders of magnitude*. Orders of magnitude information presumes partial numerical information. Hence, it is assumed that landmark values are not just symbolic anymore, but that they have associated a *numerical interval*, which specifies an *inaccurate real value*. Data manipulations and calculations are thus processed by using interval algebra as a particular qualitative algebra (see section 4.1).

6.1.1 Duration evaluation

Without mathematical integration of differential equations there is no way to give a mathemetical expression of time as a function of other parameters of the system. Classical formulas of functional analysis (Taylor) are therefore necessary to make time explicit. Notice that numerical simulation schemes are based on the same principle.

Let $x(t)$ be a variable of the system, assumed to be a continuously differentiable function of time, and consider two points t_1 and t_2, then, the first order Taylor–Lagrange formula shows that there exists t between t_1 and t_2 such that

$$x(t_2) = x(t_1) + (t_2 - t_1) \cdot dx(t)/dt$$

The duration corresponding to the state in which the variable x evolves between two consecutive landmarks x_1 and x_2 is evaluated by the formula

$$\Delta t \approx (x2 - x1)/(dx/dt)$$

where dx/dt represents the qualitative value (given by a numerical interval) of the derivative between t_1 and t_2, $\approx$ is the qualitative equality (see section 4.1), and Δt the qualitative value of the duration. Therefore, the above formula allows determination of a qualitative value of Δt from qualitative values of the landmarks x_1 and x_2, and the derivative dx/dt. Obviously, the more important the variation of the derivative in the interval $]t_1,t_2[$, the less accurate the duration. Although the formalism is slightly different here, the principle is the same as in Shen & Leitch (1990) and Kuipers & Berleant (1988). Calculus is based on the mean value theorem.

If dx/dt does not appear as a system variable, its expression may be derived analytically from the set of constraints. Orders of magnitude of the first order derivatives can be obtained by using the method in Kuipers & Chiu (1987).

6.1.2 Duration evaluation at critical points

Now, consider the cases of critical points such that:

- dx/dt reaches 0 when x reaches some landmark x_2;
- dx/dt reaches 0 when x is between two landmarks x_1 and x_2, in which case QSIM generates a new landmark x^* at a new time point t^*.

Duration calculus is then inefficient with the first order formula. Indeed, having zero as one of the limits of the numerical interval given for the derivative leads to one infinite limit for the duration.

As landmarks often correspond to the derivative changing direction (hence being zero for the landmark), and that new landmark creation is quite common as well, this situation often occurs. Hence, using the first order formula method as in Kuipers and Berleant (1988) and Shen & Leitch (1990) is not satisfactory.

Now, by using the same procedure as for first order derivatives, an analytical expression of second order derivatives can be obtained, and then used in second order Taylor–Lagrange formula

Let $x(t)$ be a variable of the system, assumed to be a continuously differentiable function of time, and consider two time points t_1 and t_2, then there exists t between t_1 and t_2 such that

$$x(t_2) = x(t_1) + (t_2 - t_1) \cdot (dx(t_1)/dt) + 1/2 \cdot (t_2 - t_1)^2 \cdot (d^2x(t)/dt^2)$$

As no assumption on whether t_1 is inferior or superior to t_2 has been made, the above formula can be used taking either t_1 or t_2, as the critical point. There is evidence that it will result in a simpler qualitative calculus (and hence more accurate) if taking t_1. This way, and since it implies that $dx(t_1)/dt = 0$, we get

$$x(t_2) = x(t_1) + 1/2 \cdot (t_2 - t_1)^2 \cdot (d^2x(t)/dt^2)$$

Let us set $\Delta x = x(t_2) - x(t_1)$ and $\Delta t = t_2 - t_1$; then two formulae have now to be considered depending on the relative position of t_1 with respect to t_2

$$t_2 < t_1 : \Delta x \approx -1/2 \cdot \Delta t^2 \cdot (d^2x(t)/dt^2)$$
$$t_2 > t_1 : \Delta x \approx +1/2 \cdot \Delta t^2 \cdot (d^2x(t)/dt^2)$$

The first case, in which there is no new landmark creation, is certainly the simplest as the only unknown variable is Δt. On the contrary, in the case of a new landmark creation, there are two unknown variables, Δx and Δt, that contain the undetermined qualitative values x^* and t^*. Hence, a second equation is required, which can be taken as the first order derivative constraint on dx/dt

$$\Delta(dx/dt) \approx \Delta t \cdot (d^2x(t)/dt^2)$$

An evaluation of Δt and also of x^*, the extremum of x at critical time point t^*, is then possible by using the two above equations.

The case of second order critical points (dx/dt and $d^2x(t)/dt^2$ simultaneously equal to zero) have not been considered here, and would require a third order approach. As a matter of fact, the above method could be extended to nth order critical points.

The major problem of the method lies on the fact that the expression of $d^2x(t)/dt^2$ generally contains the unknown variable $x_1 = x(t_1)$. This understands algebraic manipulations which may imply using a formal calculus system. The relationship between $d^2x(t)/dt^2$ and x_1 depends on the set of constraints.

6.1.3 Temporal filtering

As in Shen & Leitch (1990) and Kuipers & Berleant (1988), the method for temporal filtering is based on the following principle: given the qualitative state duration Δt_x for each variable x which might change state on the next time point, temporal filtering consists in eliminating all the transitions on the variables y for which there exists x, such that $\Delta t_x < \Delta t_y$ and Δt_x *does not intersect* Δt_y.

6.2 Causal simulation: qualitative automata

This approach (Bousson & Travé-Massuyès, 1992a) is inspired by automata theory. Observing that qualitative variables have a finite discrete quantity space, the basic idea is to consider every qualitative variable as a qualitative automaton changing state at discrete instants of time. These instants, which are denoted kT or t_k ($k = 1,2,\ldots$), are given by a reference clock.

Like in QTF and PF-based approaches, a dynamic system, is represented by an oriented graph whose nodes are relevant variables, and whose arcs are causal influences between variables. A qualitative automaton being associated to every variable, the model thus takes the form of a qualitative causal network, which can itself be viewed as an aggregated qualitative automaton.

As far as temporal aspects are concerned, the originality of this approach stands on the use of a reference clock. The clock performs periodic sampling of time, and the evolution of the system is determined by qualitative automata state changes at every sampling instant. Between two sampling instants, automata states are assumed to be steady. The frequency of the clock must be sufficiently high to be able to follow the quickest automata motions.

The linear time scale allows state duration and event dates to be easily evaluated. On the other hand, advantages of event-driven approaches are lost, but a good choice of the sampling period may greatly minimize no-change sequences.

A qualitative automaton A_i is defined as a 4-tuple $Q_A = (E,S,f,g)$ where

- E is defined as the *input set* constituted by the set of influence values acting on A_i;
- S is the *quality space* including the potential qualitative values for A_i's state;
- f is a mapping from E into S which enables to sum up the inputs;
- g is a mapping from SxS into S which updates A_i's state.

Within the causal network, qualitative automata are related by *causal relations* $R(A_e,A_r,C)$, where A_e is the emitting automaton, A_r is the receiver automaton, and C is an *activation conditions set* which indicates the conditions to be fulfilled to make active the causality link. Besides, the *direction* of a causal relation is given by the orientation of the supporting arc, and the *status* may be either *static* or *dynamic*. Other arguments can also be defined like the order of magnitude of the influence, its duration etc.

Causal relations may be ot two types: predicate or correspondence. Predicates express a compartmental constraint between the output of the emitting automaton and the input of the receiver. The operators *DERIV, M+, M−*, etc., as in the QDE approach, are examples of predicate-type relations. Correspondences are functions which make a one-to-one correspondence between the states of related automata. These latter relations have always static status.

A causal relation may be the conjunction of several relations. In this case, $R(A_e,A_r,C) = R_1(A_e,A_r,C_1) \vee \ldots \vee R_n(A_e,A_r,C_n)$ with $C_i \neq C_j$ for all $i \neq j$. It is, for example, the case of a causal relation which may change status under certain conditions. Such a relation is said to be *composite*.

Automata dynamics are defined by a *dynamic law* and a *delay time*. Delay time is an attribute which indicates the time lag between emission of the cause and appearance of the effect. For one automata, it may be related to either one ingoing arc or to a set of arcs whose effects combine. Automata dynamic laws are of the form

$$A(t+1) = g(f(e,t),A(t))$$

where $A(t)$ is the state of the automaton at instant t and $f(e,t)$ is the *global input*.

Dynamic laws may be given by composition tables which make explicit the functional relation

$$g(f(e,t),A(t)) = f(e,t) \oplus A(t)$$

The global input $f(e,t)$ of an automaton A is obtained by combining the set of received influences. Combination laws are generally obtained from expert-knowledge. In practice, experts rarely combine more than two influences. Hence, combination laws can often be given by tables as well.

The influence combination issue is still under investigation. Indeed, dialogue with experts is often laborious. There is no doubt that it would significantly gain from being more formalized. An approach based on the definition of a restricted set of basic operators, each capturing a standard combination law of the application domain and from which any complex combination could be derived, is presently being investigated (Bousson & Travé-Massuyès, 1992b). The idea is to associate, in agreeement with the expert, precise semantics to basic operators so as to be able to encode expert-knowledge more easily.

As far as prediction is concerned, simulation begins with an initial state providing the qualitative states of some automata in the graph. Then, the basic cycle during a sampling period, includes two steps

- instantaneous influence propagation at time t_k;
- automata state updating at time t_{k+1}.

So far, the qualitative automata causal network approach has been applied to prediction of future system behaviour only. However, the formalism has been thought to be suitable for explanation as well. In this case, the states of some automata will be initialized with observed data arising from process sensors and information-based causal relations will be considered to allow fast interpretations.

7 Conclusions

Qualitative reasoning reflects the motivations of several scientific communities which contemplated qualitative techniques from the sixties up to the present day. Over the last decade, artificial intelligence has provided new environments and tools for these techniques to blossom. This paper has highlighted recent advances having origins in two major contributions of AI

- the introduction of the *dynamic dimension*;
- the idea of *explanation*.

Qualitative reasoning took a radical turn when qualitative simulation algortihms were provided. The same goes for the introduction of the concept of causality which plays a major part in the explanation.

In this paper, qualitative simulation is positioned with respect to conventional simulation methods, and it is shown that it includes some aspects of discrete-event simulation, which provides advantages and limitations. The event-driven approach is undoubtedly attractive with regard to computation time and memory requirements. However, a pure event-based approach may be restrictive in many problems which require situation assessment based on state duration and event dates. Some recent investigations are presented at the end of the paper; an alternative solution for obtaining and taking advantage of temporal information in QSIM-like algorithms has been presented. On the other hand, an approach based on qualitative automata is proposed to provide causal model-based simulation algorithms with a temporal dimension.

As far as causality is concerned, it is the author's opinion that relationships with other research domains are too scarce, and that they should be investigated in the near future, this creating a new research direction. Indeed a multitude of new prospects are opening up today. It is enough to mention

- handling both qualitative and quantitive data;
- making heuristic, qualitative and quantitative models cooperate;
- integrating qualitative and quantitative techniques;

which make clear that the current trends are to allow the problem solver to use any form of available knowledge, and to integrate any method which could efficiently contribute to the solution.

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