interesting aspects are analysed, among them the entropy of fuzzy sets and probability as a special case of fuzziness.

The utility of fuzzy adaptive maps is demonstrated on the familiar control problems of backing up a truck, and truck-and-trailer, to a loading dock in a parking lot. The same methodology is then demonstrated on the tasks of real-time target tracking and signal processing.

The book is accompanied by neural-network and fuzzy-system software to enable the reader to develop practical experience with the relevant theoretical frameworks. Moreover, the author suggests to the reader many interesting problems for further consideration. Each of the individual chapters is followed by an extensive list of related literature.

Neural Networks and Fuzzy Systems is conceived of as an advanced reading, and certainly is not addressed to newcomers. The author's ambition to present his ideas in precise mathematical terms entails a highly technical style that makes the text difficult for readers without the necessary background. However, graduate students and researchers from respective fields should not fail to include the book in their library.

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First steps in modal logic by Sally Popkorn, Cambridge University Press, 1994, pp 314, £25.00, ISBN 0-521-46482-X.

If someone wants to teach themselves the basics of modal logic, three textbooks come immediately to mind. By Hughes and Cresswell there is *An introduction to Modal Logic*, and the later *A Companion to Modal Logic*; and by Chellas, *Modal Logic, an Introduction*. The most recent of these (the second Hughes and Cresswell book) was published in 1984. Given that modal logic is a rapidly maturing discipline, both in scope of its application and understanding of its foundations, it is not too surprising that a fourth introductory text has now been written which effectively complements and updates the earlier texts.

Just to recapitulate a little to set the scene, in certain contexts it has been thought valuable to enrich the language of classical logic with modalities, initially in an attempt to study the informal modalities which are used in natural language. The first to be introduced is usually $\Box$. Possible reading of $\Box\phi$ include "ϕ is necessary", "ϕ is obligatory", or "ϕ is known". The different readings suggest different semantics and proof systems. There is a close analogy between $\Box$ and universal quantification in predicate logic. However, whereas in predicate logic quantification is over variables, $\forall$x.P(x) for example being read as "for all values of x, P(x) is true", in modal logic the quantification is implicitly over contexts. So we may alternatively gloss $\Box\phi$ as "in all possible contexts ϕ is true", or "in all possible situations ϕ must be acted upon", or possibly "ϕ is true and always will be true" if the modality is used in a tense logic.

A dual statement, $\neg\Box\neg\phi$, is of sufficient importance that an additional symbol is introduced as an abbreviation: $\Diamond$. Readings of $\Diamond\phi$ include "ϕ is possible", "ϕ is permissible" and "ϕ will be true at least once". This modality has an analogy with existential quantification $\exists$; "there is a possible context in which ϕ is true", or "there is a possible situation in which ϕ is acted upon".

Having defined the basic modalities, it is then necessary to say how new statements may be derived from a logical theory which includes modal statements. Additional axioms to those of classical logic are required to enable interactions between the modalities to be modelled. For example, in a logic of knowledge and belief (where $\Box\phi$ is read as "ϕ is known"), it is usual to include the axiom: $\Box\phi \Rightarrow \Diamond\phi$. This accounts for consistency of knowledge; if the equivalent $\neg\Box\neg\phi$ is substituted for $\Diamond\phi$, this axiom can be read as "if ϕ is known, then $\neg\phi$ cannot be known".

As mentioned earlier, modal logic has been used as a basis for tense, or temporal, logics. To be sufficiently expressive, two different forms of the $\Box$ modality need to be introduced. $[+]\phi$ is usually associated with the above reading of "is and always will be", whilst $[-]\phi$ corresponds to "is and always was". Similarly, complementary forms of $\Diamond$ can be defined: $\langle + \rangle \phi$ for will be (at least once in the future), and $\langle - \rangle \phi$ for was (at least once in the past). This is now a *bimodal* system, as opposed to the monomodal system in which $\Box$ is taken as the primitive modality. One can go further. Although tense logic was originally conceived for the analysis of tenses in natural language, they are being increasingly enriched and used to analyse the behaviour of computer programs and the state transitions of finite automata. Such applications take one into the realm of linear and branching time temporal logics. Here it is necessary to add several (possibly an infinity of) new connectives $[i]$, one for each element i of an index set I. Such *polymodal* systems are not discussed in any of the textbooks referred to earlier, but are the basis for the presentation in *First Steps in Modal Logic*.

The introduction of modalities allows meta-level statements to be made within the language of the logic. This is a significant enrichment of the expressive power of the logic. Not surprisingly, the model-theoretic semantics of predicate logic needs to be enriched to capture the semantics of modal logics.

The introduction of *Kripke structures* in the 1960s provided the machinery for defining the semantics of a modal logic. As is well known, a Kripke structure consists of a set of elements, possible worlds, and a distinguished relation between the possible worlds. Essentially, $\Box\phi$ is valid if ϕ is true in all worlds accessible from the present world, $\Diamond\phi$ is valid if there exists a world accessible from the present world in which ϕ is true. The properties of the accessibility relation determine the precise variant of modal logic.

Kripke structures are one example of the more general *labelled transition structures*. Labelled transition structures consist of a non-empty set, and an I-indexed set of relations on that set. A Kripke structure corresponds to the case when I is a singleton. In the more general form, these structures provide the tools to support the semantics of polymodal logics. Popkorn's contention is that the "main objective of modal logic is, no more and no less, the study of labelled transition structures" (p. 25). The syntax of the different formal modal system and their rules of inference are, to her, merely tools to assist in the study of the properties of specific labelled transition structures. That is because, as with all the structures in model theory, labelled transition structures are direct abstractions of objects of interest (partial orderings, equivalence relations, graphs, automata and process algebras, for example) in mathematics and computing science.

First Steps in Modal Logic focuses, as a result, on the semantics of modal logic. It requires a reasonable degree of mathematical sophistication of the reader, although does not presume too much in the way of grounding in mathematical logic. Because the book concentrates on the very essence of the subject, there is no particular bias towards any specific class of users of modal logic (although Sally Popkorn admits to having computer scientists very much in mind as a target audience). An obvious advantage of this is that the book contains no unnecessary baggage. The disadvantage is that some may find the high level of abstraction hard to persist with, without a little more grounding in real world examples.

Sally Popkorn has a succinct and elegant writing style, however, and the book is a pleasure to work with. It is well structured, and there is a clear logical development of the subject matter. Each chapter concludes with examples which are designed to deepen understanding of the preceding material. Worked answers to many of these examples are collected in an appendix, which greatly increases the value of the book for self-study.

To keep the book down to a workable and affordable size, only propositional modal logic is covered. This seems a natural way of circumscribing the scope of the book, whilst still allowing the reader to gain a thorough grounding in the foundations of the subject. Pointers to the literature are provided for those who wish to develop their understanding further.

Overall, a valuable book which conveys with admirable clarity the mathematical maturity which the study of modal logic has now achieved.

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