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Modeling knowledge dynamics in multi-agent systems based on informants

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Abstract

In this paper, we model knowledge dynamics in agents' belief bases in a collaborative multi-agent system (MAS). Four change operators are introduced: expansion, contraction, prioritized revision, and non-prioritized revision. For all of them, both constructive definitions and an axiomatic characterization by representation theorems are given. We formally justify minimal change, consistency maintenance, and non-prioritization principles. These operators are based on an epistemic model for multi-source belief revision in which a rational way to weigh the beliefs using a credibility order among agents is developed. The defined operators can be seen as skills added to the agents improving the collective reasoning of a MAS.

1 Introduction and motivation

Belief revision (BR) is the process of changing beliefs to take into account a new piece of information, observation, or evidence. The AGM paradigm (Alchourrón *et al.*, 1985) has been widely accepted as a standard framework for BR; usually, *Individual Belief Revision* (IBR) in a single-agent environment is achieved satisfying or adapting AGM postulates.

Single-agent systems have evolved into multi-agent systems (MAS), where multiple interacting agents can collaborate, negotiate, discuss, etc., in order to achieve their goals. In many multi-agent domains and applications, each agent has its own initial beliefs as well as beliefs acquired from other informant agents. Hence, an agent can receive information from other agents which is contradictory with its own current beliefs. Therefore, IBR needs to be extended to multi-agent environments.

This paper focuses on multi-source belief revision (MSBR); that is, BR performed by a single agent that can obtain new beliefs from multiple informants. Therefore, one of the contributions of our approach is the definition of an epistemic model for MSBR that considers both beliefs and meta-information representing the credibility of the belief's source. We investigate how the belief base of an agent can be rationally modified when the agent receives information from other agents that can have different degrees of credibility. Thus, our main contribution is the definition based on the AGM model of four different belief change operators which use the credibility of informant agents in order to decide prevailing information. These operators are defined through constructive models and representation theorems that provide a complete axiomatic characterization for the proposed formalism.

An analysis of BR in MAS was developed in Liu & Williams (1999) where the hierarchy of Figure 1 was introduced. There, the distinction of Multi-Agent Belief Revision (MABR), MSBR and Single-agent Belief Revision (SBR) is clearly explained. In contrast to MSBR, MABR investigates the overall BR behavior of agent teams or a society in which, in order to pursue the

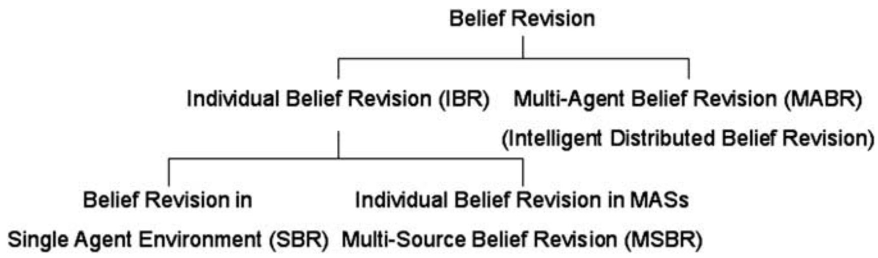

Figure 1 Belief Revision Hierarchy.

mutual goal, the agents involved need to communicate, cooperate, coordinate, and negotiate with one another. A MABR system is a MAS whose mutual goal involves BR. Different formalisms have been presented to deal with MABR (Malheiro *et al.*, 1994; Kfir-Dahav & Tennenholz, 1996; Liu & Williams, 1999, 2001). Nevertheless, in MSBR, an IBR process is carried out in a multi-agent environment where the new information may come from multiple sources that may be in conflict. The main focus of our work is this kind of BR, also studied by Dragoni *et al.* (1994).

Consider, for instance, the following (simplified) scenario. An agent A has three informants A_1, A_2 and A_3 where, for A, A_3 is the most credible and the other two are equally credible. Agent A wants to use a resource S, but it believes that S is not available. Then A_1 informs A that S is available and hence, A revises its beliefs to take into account this new piece of information. If then A_3 informs A that S is not available, since A_3 is more credible than A_2, then A should change its belief about S. Observe that new information could be rejected if the new belief is informed by a less credible agent.

In the literature, there are several studied prioritized methods (e.g. *partial meet revision* (Alchourrón *et al.*, 1985) and *kernel revision* (Hansson, 1999)). In these methods, the new information has priority over the beliefs in the base of the receiver agent. However, as is mentioned in Fermé and Hansson (1999) and Falappa *et al.* (2002), in some scenarios a prioritized method can be unrealistic. Thus, some models of BR have been developed allowing for two options: either the new information is fully accepted or completely rejected (Hansson, 1997; Makinson, 1997; Hansson *et al.*, 2001; Konieczny *et al.*, 2010). For instance, if information comes from different sources, and these sources are not equally credible, a non-prioritized method can be more adequate. In contrast, if an agent always acquires information from the same source, then a prioritized method can be used.

In this paper, based on kernel revision, we develop a complete change model for MSBR where both non-prioritized and prioritized BR are defined. First, we propose a formalism for knowledge representation in a MAS; and then, based on this formalism we define different change operators for MSBR, either to add beliefs (expansion), to withdraw beliefs (contraction), or to maintain consistency (revision). We will introduce a credibility order among agents and, based on this order, a comparison criterion among beliefs is defined. In the revision process, if inconsistency arises, the credibility order is used to decide which information prevails. The contraction operator is based on kernel contraction (Hansson, 1994) and also uses the credibility order to decide which information prevails. We show that the proposed non-prioritized revision operator satisfies the minimal change principle, and incoming information can be rejected when the agent has more credible beliefs that contradict the new information. In the literature, there are other approaches that also attach information to agents' beliefs that represents its credibility: Benferhat *et al.* (1993), Dragoni *et al.* (1994), Cantwell (1998), and Benferhat *et al.* (2002). However, our approach differs from them as we will explain in detail below.

Some preliminary works related to this paper have been reported in two workshop papers (Tamargo *et al.*, 2008, 2009). However, here, we extend both in several ways. We will define different operators which describe a complete change model based on informants: expansion, contraction, prioritized revision, and non-prioritized revision. These operators are based on an epistemic model in which a rational way to weigh beliefs is developed. These operators can be seen as skills added to the agents which improve the collective reasoning of a MAS. For these operators we give both constructive definitions and an axiomatic characterization of them by representation theorems. We also formally justify minimal change and consistency principles. Thus, a complete change model where both prioritized and non-prioritized BR are defined. The rest of this paper is structured as follows. Next, Section 2 introduces the epistemic model for MSBR. Section 3 develops the concept of plausibility used to decide which beliefs will be preserved or erased in change operators. Section 4 defines change operators based on informants: expansion, two kinds of contractions, and two kinds of revisions. Section 5 presents forwarding information among agents. Finally, in Section 6 conclusions are given and related works are commented. All proofs can be found the Section 7.

2 Epistemic model for Multi-Source Belief Revision

In this section, we introduce an epistemic model for MSBR which is based on informants. Then, in the following sections, we will define change operators based on agents interactions to add beliefs (expansion), to withdraw beliefs (contraction), and to revise beliefs. Note that the AGM model (Alchourrón *et al.*, 1985) represents epistemic states by means of belief sets, that is, sets of sentences closed under logical consequence. Other models use belief bases; that is, arbitrary sets of sentences (Fuhrmann, 1991; Hansson, 1992). Our epistemic model is based on an adapted version of belief bases which have additional information.

We adopt a propositional language $\mathcal{L}$ with a complete set of boolean connectives, namely $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$. Also, we assume the existence of an operator Cn that satisfies *inclusion* ($B \subseteq Cn(B)$), *iteration* ($Cn(B) = Cn(Cn(B))$), *monotonicity* (if $B \subseteq C$ then $Cn(B) \subseteq Cn(C)$), and *compactness* (if $\alpha \in Cn(B)$, then $\alpha \in Cn(B')$ for some finite subset $B' \subseteq B$) and includes the classical consequence operator. In general, we will write $\alpha \in Cn(B)$ as $B \vdash \alpha$.

When interacting, agents will incorporate the received information into their knowledge base in form of *information objects*. An information object will associate a sentence with an agent. For the identification of the individual agents we introduce a finite set of agent identifiers that is denoted as $\mathbb{A} = \{A_1, \ldots, A_n\}$.

Definition 1 (Information object) *An* information object *is a tuple $I = (\alpha, A_i)$, where α is a sentence of a propositional language $\mathcal{L}$ and $A_i \in \mathbb{A}$.*

Information objects are used to represent an agent's belief base. Observe that the agent identifier of an information object I can be used for representing the agent from which the information is received, or the agent that has generated the information. In Section 5, we will describe different criteria for forwarding information that will determine which agent identifier will be used.

Definition 2 (Belief base) *Let $\mathbb{A} = \{A_1, \ldots, A_n\}$ be a set of agent identifiers. A* belief base *of an agent A_i $(1 \leq i \leq n)$ is a set $K_{A_i} = \{I_1, \ldots, I_k\}$ containing information objects $(\alpha, A_j)(1 \leq j \leq n)$ received from other agents $(j \neq i)$ and proper beliefs $(j = i)$.*

Example 1 *Consider the set of agent identifiers $\mathbb{A} = \{A_1, A_2, A_3, A_4\}$ and the belief base of the agent A_1, $K_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \rightarrow \beta, A_4), (\omega, A_3), (\omega \rightarrow \beta, A_4), (\alpha \rightarrow \delta, A_1), (\alpha \rightarrow \delta, A_2), (\delta \rightarrow \beta, A_1), (\gamma, A_3), (\gamma, A_4), (\gamma \rightarrow \epsilon, A_2)\}$. Observe that K_{A_1} has two information objects with the sentence α; however, each one has a different agent identifier.*

The set $\mathcal{K} = 2^{\mathcal{L} \times \mathbb{A}}$ will represent all the belief bases. Next, two auxiliary functions are introduced in order to obtain the set of sentences or the set of agents that belong to a belief base $K \in \mathcal{K}$.

Definition 3 (Sentence function) *The* sentence function *Sen ($Sen : \mathcal{K} \rightarrow 2^{\mathcal{L}}$) is a function such that for a given belief base $K \in \mathcal{K}$, $Sen(K) = \{\alpha : (\alpha, A_i) \in K\}$.*

In our proposal, each agent $A \in \mathbb{A}$ will have a consistent belief base K_A. A belief base K_A is consistent if $Cn(Sen(K))$ is consistent.

Definition 4 (Agent identifier function) *The* agent identifier function Ag $(Ag : \mathcal{K} \to 2^{\mathbb{A}})$ *is a function such that for every base* $K \in \mathcal{K}$, $Ag(K) = \{A_i : (\alpha, A_i) \in K\}$.

Example 2 *Consider* K_{A_1} *of Example 1. Then,*

- $Sen(K_{A_1}) = \{\beta, \alpha, \alpha \to \beta, \omega, \omega \to \beta, \alpha \to \delta, \delta \to \beta, \gamma, \gamma \to \epsilon\}$.
- $Ag(K_{A_1}) = \{A_1, A_2, A_3, A_4\}$.

The agent identifier of an information object can be used to evaluate the truthfulness of the received information. In our approach, an *assessment* function will be used for representing the credibility each agent assigns to other agents. For defining this assessment, we assume a set of credibility labels $\mathcal{C} = \{c_1, \ldots, c_k\}$ (common to all agents) with an order $\prec_c$ such that for all c_1, c_2, $c_3 \in \mathcal{C}$: if $c_1 \prec_c c_2$ and $c_2 \prec_c c_3$ then $c_1 \prec_c c_3$; $c_1 \prec_c c_2$ or $c_2 \prec_c c_1$; $c_1 \prec_c c_1$ does not hold; and if $c_1 \prec_c c_2$, then $c_2 \prec_c c_1$ does not hold. That is, following (Hein, 2010), we assume an irreflexive total order (also known strict total order).

Definition 5 (Assessment) *Let* $\mathbb{A} = \{A_1, \ldots, A_n\}$ *be a set of agent identifiers and* $\mathcal{C} = \{c_1, \ldots, c_k\}$ *a set of credibility labels. An assessment* c_{A_i} *for the agent* A_i *is a function* $c_{A_i} : \mathbb{A} \to \mathcal{C}$ *assigning a credibility value from* $\mathcal{C}$ *to each agent* $A_j \in \mathbb{A}$.

The set of credibility labels is the same for all agents; however, each agent will have its own assessment and different agents may have different assessments. Thus, the assessment of an agent can be replaced in a modular way without changing its belief base and without affecting other agents assessments.

Example 3 *Consider the set of agent identifiers* $\mathbb{A} = \{A_1, A_2, A_3, A_4\}$ *and the set of credibility labels* $\mathcal{C} = \{c_1, c_2, c_3, c_4, c_5, c_6\}$, *where* $c_1 \prec_c c_2 \prec_c c_3 \prec_c c_4 \prec_c c_5 \prec_c c_6$. *The agents of* $\mathbb{A}$ *can have the following assessments:*

$A_1 : c_{A_1}(A_1) = c_1$, $c_{A_1}(A_2) = c_2$, $c_{A_1}(A_3) = c_2$ and $c_{A_1}(A_4) = c_3$.
$A_2 : c_{A_2}(A_1) = c_2$, $c_{A_2}(A_2) = c_2$, $c_{A_2}(A_3) = c_2$ and $c_{A_2}(A_4) = c_2$.
$A_3 : c_{A_3}(A_1) = c_4$, $c_{A_3}(A_2) = c_3$, $c_{A_3}(A_3) = c_2$ and $c_{A_3}(A_4) = c_1$.
$A_4 : c_{A_4}(A_1) = c_4$, $c_{A_4}(A_2) = c_3$, $c_{A_4}(A_3) = c_2$ and $c_{A_4}(A_4) = c_1$.

Observe that for agent A_2 *all agents have the same credibility, for agent* A_3 *all agents have different credibility, and agents* A_3 *and* A_4 *have the same assessment.*

Thus, based on its own assessment, each agent can have a credibility order over the set $\mathbb{A}$.

Definition 6 (Credibility order among agents) *A* credibility order among agents *for an agent* A_i, *denoted by* '$\leq_{Co}^{A_i}$', *is a total order over* $\mathbb{A}$ *where* $A_1 \leq_{Co}^{A_i} A_2$ *means that according to* A_i, A_2 *is at least as credible than* A_1, *and holds if* $c_{A_i}(A_1) \prec_c c_{A_i}(A_2)$ *or* $c_{A_i}(A_1) = c_{A_i}(A_2)$. *The strict relation* $A_1 <_{Co}^{A_i} A_2$, *denoting that* A_2 *is strictly more credible than* A_1, *is defined as* $A_1 \leq_{Co}^{A_i} A_2$ *and* $A_2 \not\leq_{Co}^{A_i} A_1$. *Moreover,* $A_1 =_{Co}^{A_i} A_2$ *means that* A_1 *is as credible as* A_2, *and it holds when* $A_1 \leq_{Co}^{A_i} A_2$ *and* $A_2 \leq_{Co}^{A_i} A_1$.

Since '$\leq_{Co}^{A_i}$', is a total order over $\mathbb{A}$ then for all $A_1, A_2, A_3 \in \mathbb{A}$ it holds:

- Reflexive: $A_1 \leq_{Co}^{A_i} A_1$.
- Totality or Completeness: $A_1 \leq_{Co}^{A_i} A_2$ or $A_2 \leq_{Co}^{A_i} A_1$.
- Transitivity: if $A_1 \leq_{Co}^{A_i} A_2$ and $A_2 \leq_{Co}^{A_i} A_3$, then $A_1 \leq_{Co}^{A_i} A_3$.
- Antisymmetry: if $A_1 \leq_{Co}^{A_i} A_2$ and $A_2 \leq_{Co}^{A_i} A_1$, then $A_1 =_{Co}^{A_i} A_2$.

Example 4 *Consider the set of agent identifiers* $\mathbb{A} = \{A_1, A_2, A_3, A_4\}$ *and the set of credibility labels* $\mathcal{C} = \{c_1, c_2, c_3\}$, *where* $c_1 \prec_c c_2 \prec_c c_3$. *Suppose that according to the assessment of* A_1, $c_{A_1}(A_1) = c_1$, $c_{A_1}(A_2) = c_2$, $c_{A_1}(A_3) = c_2$ *and* $c_{A_1}(A_4) = c_3$. *Then, the credibility order, according to* A_1, *is:* $A_1 \leq_{Co}^{A_1} A_2, A_1 \leq_{Co}^{A_1} A_3, A_1 \leq_{Co}^{A_1} A_4, A_2 \leq_{Co}^{A_1} A_3, A_3 \leq_{Co}^{A_1} A_2, A_2 \leq_{Co}^{A_1} A_4$, *and* $A_3 \leq_{Co}^{A_1} A_4$. *Hence,* $A_1 <_{Co}^{A_1} A_2 =_{Co}^{A_1} A_3 <_{Co}^{A_1} A_4$.

The information received by an agent can be contradictory with its current beliefs. For instance, consider again the belief base (K_{A_1}) of Example 1, where $Sen(K_{A_1}) \vdash \beta$ (observe that there are several derivations for β). Suppose now that the agent A_1 receives the information object $I = (\neg\beta, A_4)$. It is clear that adding $(\neg\beta, A_4)$ to K_{A_1} will produce an inconsistent belief base. Therefore, the agent has to decide whether it rejects $(\neg\beta, A_4)$ or withdraws β. In our approach, the credibility order '$\leq_{Co}^{A_i}$' will be used to decide which information prevails. If the new incoming information prevails, then the agent has to withdraw β. To do that, an adapted version of *Kernel contractions* will be introduced where all the minimal subsets of K_{A_1} that entail β will be considered.

Kernel contractions were introduced in (Hansson, 1994) and they are based on a selection among the sentences that are relevant to derive the sentence to be retracted. Note that kernel contractions are a generalization of *safe contractions* proposed in (Alchourrón and Makinson, 1985). In order to perform a contraction, kernel contractions use incision functions which cut into the minimal subsets that entail the information to be given up. Therefore, we will adapt the definition of α-kernel to our epistemic model which will be used below to define a comparison criterion among sentences (called plausibility) and to define incision functions.

Definition 7 (α-kernel) *Let $K \in \mathcal{K}$ and $\alpha \in \mathcal{L}$. Then H is an α-kernel of K if and only if*

1 $H \subseteq K$.
2 $Sen(H) \vdash \alpha$.
3 *if* $H' \subset H$, *then* $Sen(H') \nvdash \alpha$.

Note that an α-*kernel* is a minimal set of tuples from K that entails α. The set of α-*kernel* of K is denoted $K^{\perp\!\!\!\perp}\alpha$ and is called *kernel set* (Hansson, 1994).

Example 5 *Consider K_{A_1} of Example 1.*
$K_{A_1}{}^{\perp\!\!\!\perp}\beta = \{H_a, H_b, H_c, H_d, H_e, H_f, H_g, H_h\}$ *where*

$$
\begin{array}{ll}
H_a = \{(\beta, A_1)\} & H_e = \{(\alpha, A_2), (\alpha \to \delta, A_1), (\delta \to \beta, A_1)\} \\
H_b = \{(\alpha, A_2), (\alpha \to \beta, A_4)\} & H_f = \{(\alpha, A_3), (\alpha \to \delta, A_1), (\delta \to \beta, A_1)\} \\
H_c = \{(\alpha, A_3), (\alpha \to \beta, A_4)\} & H_g = \{(\alpha, A_2), (\alpha \to \delta, A_2), (\delta \to \beta, A_1)\} \\
H_d = \{(\omega, A_3), (\omega \to \beta, A_4)\} & H_h = \{(\alpha, A_3), (\alpha \to \delta, A_2), (\delta \to \beta, A_1)\}
\end{array}
$$

Observe that a belief base can contain the same sentence in two (or more) information objects with different agent identifiers. For instance, in Example 1 $\{(\alpha, A_2), (\alpha, A_3)\} \subseteq K_{A_1}$. As it will be explained in detail below, when an agent A_i receives an information object (α, A_j) consistent with its current belief base (*i.e.*, $Sen(K_{A_i}) \nvdash \neg\alpha$), then (α, A_j) is added to K_{A_i} (expansion). Note that it may be the case that $Sen(K_{A_i}) \vdash \alpha$; however, (α, A_j) is also added to K_{A_i} because the credibility of the associated agent can increase the plausibility of α. From the information objects point of view, there is no redundancy due to the fact that each information object represents a different informant.

Thus, in a belief base the same sentence can be in several information objects (with different agent identifiers). Therefore, if the assessment of the agent is changed and the credibility of a particular agent is increased, then all the sentences associated to this agent automatically have more credibility. Nevertheless, it will be useful that given a belief base K, a compacted belief base K' can be obtained; that is, a base where there are no tuples with the same sentence and the more credible associated agent remains. In order to make more efficient the construction of changes, we propose that kernel sets can be computed over compacted belief bases.

Next, a function that given a belief base returns a compacted one is introduced (Definition 10). This function needs to know which is the most credible associated agent with respect to a given sentence, and this is returned by the *top agent function*.

Definition 8 (Top agent function) *The* top agent function, $Top : \mathcal{L} \times \mathcal{K} \to 2^{\mathbb{A}}$, *is a function such that for a given belief base $K_{A_i} \in \mathcal{K}$ and a given sentence $\alpha \in Sen(K_{A_i})$, $Top(\alpha, K_{A_i}) = \{A_k : (\alpha, A_k) \in K_{A_i}$ and for all $(\alpha, A_j) \in K_{A_i}, A_j \leq_{Co}^{A_i} A_k\}$.*

We assume that there is a function (see Definition 9) that based on a given policy[1] returns a single-agent identifier from a set of agent identifiers to which the assessment assigns the same label. For instance, the policy could be based on a lexicographical ordering among agent identifiers -A_1 is lesser than A_2.

Definition 9 (Selection function) *The* selection function *of an agent A_i, $\mathcal{S}_{A_i} : 2^{\mathbb{A}} \to \mathbb{A}$, is a function such that for a given set of agent identifiers with equal credibility with respect to the assessment of A_i, it returns a single-agent identifier based on a given policy.*

Definition 10 (Compact belief base function) *The* compact belief base function *($Compact : \mathcal{K} \to \mathcal{K}$) is a function such that for a given belief base $K_{A_i} \in \mathcal{K}$:*

$$Compact(K_{A_i}) = \{(\alpha, A_j) : (\alpha, A_j) \in K_{A_i} \text{ and } A_j = \mathcal{S}_{A_i}(Top(\alpha, K_{A_i}))\}$$

In order to simplify the notation we use $K^{\uparrow}_{A_i}$ instead of $Compact(K_{A_i})$.

Example 6 *Consider again the agent A_1 of Example 1, where $K_{A_1}\{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \to \beta, A_4), (\omega, A_3), (\omega \to \beta, A_4), (\alpha \to \delta, A_1), (\alpha \to \delta, A_2), (\delta \to \beta, A_1), (\gamma, A_3), (\gamma, A_4), (\gamma \to \epsilon, A_2)\}$, and consider the credibility order among agents according to A_1 from Example 4, $A_1 <^{A_1}_{Co} A_2 =^{A_1}_{Co} A_3 <^{A_1}_{Co} A_4$. Then,*

- *$Top(\gamma, K_{A_1}) = \{A_4\}$ and $Top(\alpha \to \delta, K_{A_1}) = \{A_2\}$.*
- *$Top(\alpha, K_{A_1}) = \{A_2, A_3\}$.*
- *$\mathcal{S}_{A_1}(\{A_2, A_3\}) = A_2$ where the policy adopted is based on a lexicographical ordering among agent identifiers.*
- *The compact belief base is:*

$$\begin{aligned} K^{\uparrow}_{A_1} =& \{(\beta, A_1), (\alpha, A_2), (\alpha \to \beta, A_4), (\omega, A_3), (\omega \to \beta, A_4), \} \cup \\ & \cup \{(\alpha \to \delta, A_2), (\delta \to \beta, A_1), (\gamma, A_4), (\gamma \to \epsilon, A_2)\} \end{aligned}$$

- *$K^{\uparrow}_{A_1} \perp\!\!\!\perp \beta = \{H_a, H_b, H_d, H_g\}$ where*

$$\begin{aligned} H_a &= \{(\beta, A_1)\} & H_d &= \{(\omega, A_3), (\omega \to \beta, A_4)\} \\ H_b &= \{(\alpha, A_2), (\alpha \to \beta, A_4)\} & H_g &= \{(\alpha, A_2), (\alpha \to \delta, A_2), (\delta \to \beta, A_1)\} \end{aligned}$$

It is important to note that from the Definitions 3, 7, 8 and 10, the following proposition is straightforwardly deduced.

Proposition 1 *Let $K_{A_i} \in \mathcal{K}$, it holds that: $K^{\uparrow}_{A_i} \subseteq K_{A_i}$, $Sen(K^{\uparrow}_{A_i}) = Sen(K_{A_i})$, and $K^{\uparrow}_{A_i} \perp\!\!\!\perp \alpha \subseteq K_{A_i}^{\perp\!\!\!\perp} \alpha$.*

Consider $\alpha \in \mathcal{L}$ and $\beta \in Sen(K_{A_i})$ such that β is in m tuples of K_{A_i} ($m > 1$). Then $K^{\uparrow}_{A_i} \subset K_{A_i}$. If $X \in K^{\uparrow}_{A_i} \perp\!\!\!\perp \alpha$ and $\beta \in Sen(X)$, then $K_{A_i}^{\perp\!\!\!\perp} \alpha$ will have at least m *α-kernels* differing only in the agent identifier of the tuple in which β is in. Nevertheless, since $K^{\uparrow}_{A_i}$ has only one tuple containing β, then $K^{\uparrow}_{A_i} \perp\!\!\!\perp \alpha \subset K_{A_i}^{\perp\!\!\!\perp} \alpha$. In the following section, we will prove that it is equivalent to compute the plausibility of a sentence either with $K^{\uparrow}_{A_i}$ or with K_{A_i}.

3 Sentences plausibility

As stated above, when an agent A_i receives an information object that is inconsistent with its knowledge base (e.g. it receives $(\neg\beta, A_4)$ and it holds that $Sen(K_{A_i}) \vdash \beta$), then the credibility order among agents $\leq^{A_i}_{Co}$ will be used to decide which sentence prevails. Note that a sentence can have more than one derivation from a given knowledge base. Therefore, a comparison order among sentences (called *Plausibility*) will be defined. That is, if α and β are sentences, the notation $\alpha \preceq_{K_{A_i}} \beta$ will represent the following: for the agent A_i, β is at least as plausible as α relative to its assessment c_{A_i} and its belief base K_{A_i}. The plausibility of a sentence will be used to define revision and contraction operators.

[1] A policy can be seen as a design decision.

The concept of plausibility is related to *epistemic entrenchment* (Gärdenfors and Makinson, 1988) although the epistemic entrenchment orders are structured in a very specific way, and we apply it on belief bases instead of belief sets. According to (Gärdenfors, 1992), '…some sentences in a belief system have a higher degree of epistemic entrenchment than others... The guiding idea for the construction is that when a belief set **K** is revised or contracted, the sentences in **K** that are given up are those having the lowest degrees of epistemic entrenchment'.

The following function characterizes all the sentences that can be entailed from a belief base.

Definition 11 (Belief function) *The* belief function, $Bel : \mathcal{K} \to 2^{\mathcal{L}}$, *is a function such that for a given belief base* $K \in \mathcal{K}$, $Bel(K) = \{\alpha : \alpha \in \mathcal{L} \text{ and } Sen(K) \vdash \alpha\}$.

Similar to Proposition 1, note that from the Definitions 8, 10 and 11, the following proposition is straightforwardly deduced.

Proposition 2 *Let* $K_{A_i} \in \mathcal{K}$, *it holds that* $Bel(K^{\uparrow}_{A_i}) = Bel(K_{A_i})$.

In order to calculate the plausibility of a sentence β, all its proofs have to be analyzed. Since we adopt a cautious approach, from each β-kernel we will consider those tuples that have the agent identifiers that are less credible. Two auxiliary functions are introduced below:

Definition 12 (Least credible sources function) *The* least credible sources function, $min : \mathcal{K} \to 2^{\mathcal{K}}$, *is a function such that for a given belief base* $K_{A_i} \in \mathcal{K}$, $min(K_{A_i}) = \{(\alpha, A_k) : (\alpha, A_k) \in K_{A_i}$ *and for all* $(\delta, A_j) \in K_{A_i}, A_k \leq^{A_i}_{Co} A_j\}$.

Definition 13 (Most credible sources function) *The most credible sources function*, $max : \mathcal{K} \to 2^{\mathcal{K}}$, *is a function such that for a given belief base* $K_{A_i} \in \mathcal{K}$, $max(K_{A_i}) = \{(\alpha, A_k) : (\alpha, A_k) \in K_{A_i}$ *and for all* $(\delta, A_j) \in K_{A_i}, A_j \leq^{A_i}_{Co} A_k\}$.

Example 7 *Consider the set of agent identifiers* $\mathbb{A} = \{A_1, A_2, A_3\}$ *and the credibility order of agent* A_1: $A_1 <^{A_1}_{Co} A_2 <^{A_1}_{Co} A_3$. *Let* $K_{A_1} = \{(\alpha, A_1), (\alpha, A_2), (\beta, A_1), (\gamma, A_1), (\alpha \to \gamma, A_3)\}$ *be the belief base of* A_1. *Then,*

- $min(K_{A_1}) = \{(\alpha, A_1), (\beta, A_1), (\gamma, A_1)\}$.
- $max(K_{A_1}) = \{(\alpha \to \gamma, A_3)\}$.

Next, based on the agent comparison criterion $\leq^{A_i}_{Co}$ of each agent, we will define a comparison criterion among sentences of $Bel(K_{A_i})$. First, we introduce the function $Pl(\alpha, K_{A_i})$ that given a sentence $\alpha \in Bel(K_{A_i})$, it returns an agent identifier that represents the plausibility of α with respect to the assessment of agent A_i. Then, based on the function Pl, in Definition 15, a comparison criterion $\preceq_{K_{A_i}}$ among sentences of $Bel(K_{A_i})$ is introduced.

Definition 14 (Plausibility function) *The* plausibility function, $Pl : \mathcal{L} \times \mathcal{K} \to \mathbb{A}$, *is a function such that for a given belief base* $K_{A_i} \in \mathcal{K}$ *and a sentence* $\alpha \in Bel(K_{A_i})$:

$$Pl(\alpha, K_{A_i}) = \mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K^{\uparrow \perp\!\!\!\perp}_{A_i} \alpha} min(X))))$$

Observe that the function *max* can return more than one agent identifier, therefore *Pl* uses the selection function $\mathcal{S}_{A_i}$ of Definition 9 that returns only one identifier. Note also that it may be the case in which $(\gamma, A_1) \in K_{A_1}$ and $Pl(\gamma, K_{A_1}) \neq A_1$. For instance, consider Example 7, there $Pl(\alpha, K_{A_1}) = A_2$, $Pl(\beta, K_{A_1}) = A_1$ and $Pl(\gamma, K_{A_1}) = A_2$.

Definition 15 (Plausibility criterion) *Let* $K_{A_i} \in \mathcal{K}$ *be the belief base of agent* A_i *and let* $\{\alpha, \beta\} \subseteq Bel(K_{A_i})$, *then* $\alpha \preceq_{K_{A_i}} \beta$ *if and only if it holds that* $Pl(\alpha, K_{A_i}) \leq^{A_i}_{Co} Pl(\beta, K_{A_i})$.

Thus, the notation $\alpha \preceq_{K_{A_i}} \beta$ will represent: 'for the agent A_i, β is at least as plausible as α'. The strict relation $\alpha \prec_{K_{A_i}} \beta$, representing '$\beta$ is more plausible than α', is defined as '$\alpha \preceq_{K_{A_i}} \beta$ and $\beta \not\preceq_{K_{A_i}} \alpha$'. Moreover, $\alpha \simeq_{K_{A_i}} \beta$ means that α is as plausible as β, and it holds when $\alpha \preceq_{K_{A_i}} \beta$ and $\beta \preceq_{K_{A_i}} \alpha$. From the previous definition we can observe that the plausibility of the sentences inherits the properties of the *credibility order among agents* ('$\preceq_{K_{A_i}}$' is a total order on $\mathcal{L}$). Furthermore, note that the relation '$\preceq_{K_{A_i}}$' is only defined with respect to a given K_{A_i} (different belief bases may be associated with different orderings of plausibility).

Example 8 *Consider a set* $\mathbb{A} = \{A_1, A_2, A_3\}$. *Suppose that agent* A_2 *has the following belief base* $K_{A_2} = \{(\alpha, A_1), (\beta, A_2), (\gamma, A_3)\}$ *and according to* A_2 *the credibility order is* $A_1 <^{A_2}_{Co} A_2 <^{A_2}_{Co} A_3$. *Furthermore, suppose that agent* A_3 *has the following belief base* $K_{A_3} = \{(\alpha, A_1), (\beta, A_3), (\gamma, A_2)\}$ *and the same credibility order than* A_2, $A_1 <^{A_3}_{Co} A_2 <^{A_3}_{Co} A_3$. *Then, for both agents,* β *is more plausible than* α *(i.e.* $\alpha \prec_{K_{A_2}} \beta$ *and* $\alpha \prec_{K_{A_3}} \beta$*). However, for* A_2, γ *is more plausible than* β *(*$\beta \prec_{K_{A_2}} \gamma$*) whereas for* A_3, β *is more plausible than* γ *(*$\gamma \prec_{K_{A_3}} \beta$*). In this example* A_2 *and* A_3 *have the same assessment and* $Sen(K_{A_2}) = Sen(K_{A_3})$ *but their beliefs have different associated agents. It is clear that two agents with the same belief base but different credibility orders produce different orderings of plausibility. For instance, consider that* $K_{A_1} = K_{A_2}$ *and* $A_2 <^{A_1}_{Co} A_1 <^{A_1}_{Co} A_3$ *then* $\alpha \prec_{K_{A_2}} \beta$ *but* $\beta \prec_{K_{A_1}} \alpha$.

The following example shows how the plausibility of a sentence can be calculated from a kernel set obtained from a compacted belief base.

Example 9 *Consider again Example 1, where the belief base of* A_1 *is* $K_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \to \beta, A_4), (\omega, A_3), (\omega \to \beta, A_4), (\alpha \to \delta, A_1), (\alpha \to \delta, A_2), (\delta \to \beta, A_1), (\gamma, A_3), (\gamma, A_4), (\gamma \to \epsilon, A_2)\}$, *and consider the credibility order among agents according to* A_1 *from Example 4,* $A_1 <^{A_1}_{Co} A_2 =^{A_1}_{Co} A_3 <^{A_1}_{Co} A_4$. *Then, suppose that agent* A_1 *needs to calculate the plausibility of* β. *In order to do so,* A_1 *will do the following steps.*

- Step 1. *Find the minimal subsets that derive* β *from the compacted belief base* K_{A_1} *(*$K^{\uparrow}_{A_1} \perp\!\!\!\perp \beta$*). From Example 6 we can see that:*
  $K^{\uparrow}_{A_1} \perp\!\!\!\perp \beta = \{H_a, H_b, H_d, H_g\}$.
- Step 2. *Apply 'min' to each* β*-kernel* $\in K^{\uparrow}_{A_1} \perp\!\!\!\perp \beta$:

  $min(H_a) = \{(\beta, A_1)\}$ $\quad$ $min(H_d) = \{(\omega, A_3)\}$
  $min(H_b) = \{(\alpha, A_2)\}$ $\quad$ $min(H_g) = \{(\delta \to \beta, A_1)\}$
- Step 3. *Apply 'max' to the union of all the sets found in step 2.*
  $max(\{(\beta, A_1), (\alpha, A_2), (\omega, A_3), (\delta \to \beta, A_1)\}) = \{(\alpha, A_2), (\omega, A_3)\}$.
- Step 4. *Find from the tuples of the previous item, the set containing the agent identifiers:*
  $\mathbb{A}(\{(\alpha, A_2), (\omega, A_3)\}) = \{A_2, A_3\}$.
- Step 5. *Find from the set of agent identifiers of the previous item, a single-agent identifier based on a given policy. For instance, if the policy is the lexicographical ordering among agent identifiers, then* $\mathcal{S}_{A_1}(\{A_2, A_3\}) = A_2$.

Therefore, $Pl(\beta, K_{A_1}) = A_2$.

Proposition 3 shows that given a belief base K_{A_i}, the plausibility of a sentence can be obtained from either K_{A_i} or $K^{\uparrow}_{A_i}$. However, applying the computation to K_{A_i} requires computing more kernels than with $K^{\uparrow}_{A_i}$.

Proposition 3 *Let* $K_{A_i} \in \mathcal{K}$ *and let* $\alpha \in Bel(K_{A_i})$, *then the plausibility of* α *in the belief base* K_{A_i} *is equal to the plausibility of* α *in the compacted belief base* $K^{\uparrow}_{A_i}$. *That is,*
$Pl(\alpha, K_{A_i}) = \mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K^{\uparrow}_{A_i} \perp\!\!\!\perp \alpha} min(X)))) = \mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K_{A_i} \perp\!\!\!\perp \alpha} min(X))))$.

Proof: See Appendix *in Section 7.*

Since the belief base of an agent may contain the same sentence in several different tuples, it could be natural to preserve only 'the most plausible derivation' of each sentence. However, in the following example it is shown that this criterion may be problematic.

Example 10 *Consider* $\mathbb{A} = \{A_1, A_2, A_3\}$ *where* $A_3 <^{A_2}_{Co} A_2 <^{A_2}_{Co} A_1$. *Let* $K_{A_2} = \{(\beta \to \alpha, A_2), (\alpha, A_3)\}$ *be the belief base of* A_2. *Suppose that* A_2 *incorporates* (β, A_2) *to* K_{A_2}. *In this scenario there are two derivations for* α, *and* $Pl(\alpha, K_{A_2}) = A_2$. *Note that the plausibility of* α *was increased and it is unnatural to withdraw sentences from* K_{A_2} *in order to preserve just one derivation of* α.

As we have shown in the previous example, it is very restrictive to have each sentence supported by only one derivation. For this reason, belief bases may be non-compacted. Thus, the plausibility of a sentence will be determined only by the *plausibility function*. Another reason for this decision is that we achieve a more dynamic framework since the evaluation of the credibility of the agent identifiers is separated by use of the assessment function.

It is important to note that the assessment function may change in time realizing dynamic assessments. Hence, the credibility order among agents can be changed without changing the knowledge base.

4 Change operators based on informants

In this section, we will define a change theory for MAS focusing on MSBR. The most widely studied model for BR is AGM model (Alchourrón *et al.*, 1985) which distinguishes three change operators: expansions, contractions and revisions. The AGM model represents epistemic states by means of belief sets, that is, sets of sentences closed under logical consequence. Nevertheless, as introduced above, our epistemic model uses belief bases; that is, arbitrary sets of sentences. Next, in Section 4.1 we will define an expansion operator based on our epistemic model. Then in Section 4.2, we will introduce two contraction operators. In Section 4.3, we will define a prioritized revision operator and finally, in Section 4.4 we will propose a non-prioritized revision operator.

4.1 Expansion operator based on informants

In this subsection we will define an expansion operator for our epistemic model. This is the most simple operator to characterize from the logical point of view because it consists only in the addition of new information objects.

Definition 16 (Expansion using plausibility) *Let $K_{A_i} \in \mathcal{K}$ the belief base of an agent A_i and (α, A_j) an information object. The operator '+', called* expansion using plausibility, *is defined as follows:*

$$K_{A_i} + (\alpha, A_j) = K_{A_i} \cup \{(\alpha, A_j)\}$$

In contrast to the expansion proposed in (Hansson, 1999), here we consider information objects instead of single sentences. Therefore, if $\alpha \in Bel(K_{A_i})$, then this operation could increase the plausibility of α. This operation, as Hansson's expansion, does not guarantee a consistent epistemic state.

Let $K_{A_i}, K_{A_j} \in \mathcal{K}$ two belief bases and let $A_i, A_j, A_k \in \mathbb{A}$. The expansion operator will be represented by '+'. We propose the following postulates for expansion using plausibility operator.

(EP-1) *Success:* $(\alpha, A_j) \in K_{A_i} + (\alpha, A_j)$.

The first postulate establishes that the expansion should be successful; that is, the result of expanding a belief base K_{A_i} by an information object (α, A_j) should be a new belief base that contains (α, A_j).

(EP-2) *Inclusion:* $K_{A_i} \subseteq K_{A_i} + (\alpha, A_j)$.

Obtaining information is a very expensive process, thus avoiding the unnecessary loss of information is wished in any change operator. Since $K_{A_i} + (\alpha, A_j)$ follows from adding an information object to K_{A_i} without withdrawing any belief, it is natural to think that K_{A_i} does not contain beliefs that do not belong to $K_{A_i} + (\alpha, A_j)$.

(EP-3) *Vacuity:* If $(\alpha, A_j) \in K_{A_i}$ then $K_{A_i} + (\alpha, A_j) = K_{A_i}$.

A particular case of expansion occurs when a belief base K_{A_i} is expanded by an information object (α, A_j) which is in K_{A_i}. In this case, expanding K_{A_i} by (α, A_j) does not generate any change in K_{A_i}.

(EP-4) *Monotonicity:* If $K_{A_j} \subseteq K_{A_i}$ then $K_{A_j} + (\alpha, A_k) \subseteq K_{A_i} + (\alpha, A_k)$.

Suppose that there are two belief bases and one of these is contained in the other. If both belief bases are expanded by the same belief then the inclusion relation between them should be preserved.

(EP-5) *Dynamic Plausibility:* If $\alpha \in Bel(K_{A_i})$ then $Pl(\alpha, K_{A_i}) \leq_{Co}^{A_i} Pl(\alpha, K_{A_i} + (\alpha, A_j))$.

Suppose a belief base K_{A_i} is expanded by an information object (α, A_j) where $\alpha \in Bel(K_{A_i})$. In this case, the result of expanding K_{A_i} by (α, A_j) should not decrease the plausibility of α. Thus, this operation could increase the plausibility of α.

The postulates EP-1 … EP-5 characterize axiomatically our expansion operator. For all belief base K and all information object (α, A_i), $K + (\alpha, A_i)$ is the smallest belief base which satisfies EP-1 … EP-5. Note that, EP-1 … EP-4 are defined in a similar way to those that define the expansion in AGM (Alchourron *et al.*, 1985), whereas the new postulate EP-5 considers the case that our belief base contains a belief with different associated agents.

4.2 Contraction operator based on informants

The contraction is a change operation which withdraws beliefs without adding anything. In practice, this situation occurs when an agent believes in α and perceives that $\neg\alpha$ is true. In this case, before adding $\neg\alpha$ it should be withdrawn α. Even thought this operation gives rise to an adding of a new belief, can be discomposed in two operations: a contraction with respect to α, and a subsequent expansion with respect to $\neg\alpha$. Below we introduce two contraction operators that are based on kernel contraction and adapted to our epistemic model.

4.2.1 Construction

In the belief base of an agent, several derivations for one sentence can exist. For instance, consider again the belief base (K_{A_1}) of Example 9. Suppose now agent A_1 needs to withdraw β from its belief base. Since there are several derivations of β, then it has to 'cut' all of them. The credibility order will be used to decide which information prevails. For doing that, all the minimal subsets of K_{A_1} that entail β are obtained.

Kernel contractions are based on a selection among the sentences that are relevant to derive the sentence to be retracted. In order to perform a contraction, kernel contractions use incision functions which cut into the minimal subsets that entail the information to be given up. We will adapt this notion to our epistemic model. An incision function only selects information objects that can be relevant for α and at least one element from each α-kernel.

Definition 17 (Incision function) *An* incision function σ *for* $K_{A_i} \in \mathcal{K}$ *is a function such that for all* α:

1 $\sigma(K_{A_i}^{\perp\!\!\perp}\alpha) \subseteq \cup(K_{A_i}^{\perp\!\!\perp}\alpha)$, and
2 *if* $\emptyset \neq X \in K_{A_i}^{\perp\!\!\perp}\alpha$, *then* $X \cap \sigma(K_{A_i}^{\perp\!\!\perp}\alpha) \neq \emptyset$.

In the definition of an *incision function* in Hansson's work it is not specified how the function selects the sentences that will be discarded of each α-kernel. In our approach, this will be solved with the sentences plausibility that we have defined above. Thus, the incision function will select the least credible information objects of each α-kernel.

Definition 18 (Bottom incision function) $\sigma_\downarrow$ *is a* bottom incision function *for* K_{A_i} *if* $\sigma_\downarrow$ *is an incision function such that,* $\sigma_\downarrow(K_{A_i}^{\perp\!\!\perp}\alpha) = \{(\delta, A_k) : (\delta, A_k) \in X \in K_{A_i}^{\perp\!\!\perp}\alpha$ *and for all* $(\beta, A_j) \in X$ *it holds that* $\delta \preceq_X \beta\}$[2].

Example 11 *Consider a set* $\mathbb{A} = \{A_1, A_2, A_3\}$ *where the credibility order according to* A_2 *is* $A_1 <_{Co}^{A_2} A_2 <_{Co}^{A_2} A_3$. *Suppose that the agent* A_2 *has the following belief base* $K_{A_2} = \{(\alpha, A_3),$

[2] We assume that, given a relation $\preceq_{K_{A_i}}$ on $\mathcal{L} \times \mathcal{L}$, it is possible to define a relation $\preceq_X$ on every $X \subseteq K_{A_i}$.

(β, A_2), $(\beta \to \alpha, A_1), (\beta \to \alpha, A_3), (\omega, A_1), (\omega \to \alpha, A_3), (\delta, A_1)\}$. *Then,* $K_{A_2}^{\perp\!\!\perp}\alpha = \{H_a, H_b, H_c, H_d\}$ *where:*

$$H_a = \{(\alpha, A_3)\} \qquad H_c = \{(\beta, A_2), (\beta \to \alpha, A_3)\}$$
$$H_b = \{(\beta, A_2), (\beta \to \alpha, A_1)\} \quad H_d = \{(\omega, A_1), (\omega \to \alpha, A_3)\}$$

Then, the bottom incision function is:

$$\sigma_\downarrow(K_{A_2}^{\perp\!\!\perp}\alpha) = \{(\alpha, A_3), (\beta \to \alpha, A_1), (\beta, A_2), (\omega, A_1)\}$$

Now that we have given the necessary background, two contraction operators will be defined. One of these operators (Definition 19) takes into consideration the whole belief base, and the other (Definition 20) considers its associated compacted belief base when an agent wants to apply the contraction operator.

Definition 19 (Contraction using plausibility) *Let* $K_{A_i} \in \mathcal{K}$, $\alpha \in \mathcal{L}$ *and let* $\sigma_\downarrow$ *be a bottom incision function for* K_{A_i}. *The operator '*$\ominus_{\sigma_\downarrow}$*', called contraction using plausibility, is defined as follows:*

$$K_{A_i} \ominus_{\sigma_\downarrow} \alpha = K_{A_i} \setminus \sigma_\downarrow(K_{A_i}^{\perp\!\!\perp}\alpha)$$

Note that when an agent wishes to contract its belief base for a sentence, it applies the contraction operator over the sentence and not over an information object. Furthermore, note that, it makes sense to have a version of contraction where the object to be contracted is a determined tuple (and not a sentence); however, we consider that it is not nessesary for the aim of this article.

Example 12 *Consider the set* $\mathbb{A} = \{A_1, A_2, A_3, A_4\}$ *where the credibility order according to A_1 is* $A_1 <_{Co}^{A_1} A_2 =_{Co}^{A_1} A_3 <_{Co}^{A_1} A_4$. *Suppose that the agent A_1 has the following belief base* $K_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \to \beta, A_2), (\alpha \to \beta, A_4), (\omega, A_1), (\omega \to \beta, A_4), (\alpha \to \delta, A_2), (\delta \to \beta, A_1), (\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_2), (\epsilon \to \beta, A_3), (\epsilon \to \beta, A_4)\}$. *Then, suppose A_1 wants to contract by* β *using '*$\ominus_{\sigma_\downarrow}$*'.*

- Step 1. *Find the minimal subsets that derive* β *from* K_{A_1}.
  $K_{A_1}{}^{\perp\!\!\perp}\beta = \{H_a, H_b, H_c, H_d, H_e, H_f, H_g, H_h, H_i, H_j, H_k\}$ *where*

$$H_a = \{(\beta, A_1)\} \qquad H_g = \{(\alpha, A_2), (\alpha \to \delta, A_2), (\delta \to \beta, A_1)\}$$
$$H_b = \{(\alpha, A_2), (\alpha \to \beta, A_2)\} \qquad H_h = \{(\alpha, A_3), (\alpha \to \delta, A_2), (\delta \to \beta, A_1)\}$$
$$H_c = \{(\alpha, A_3), (\alpha \to \beta, A_2)\} \qquad H_i = \{(\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_2)\}$$
$$H_d = \{(\alpha, A_2), (\alpha \to \beta, A_4)\} \qquad H_j = \{(\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_3)\}$$
$$H_e = \{(\alpha, A_3), (\alpha \to \beta, A_4)\} \qquad H_k = \{(\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_4)\}$$
$$H_f = \{(\omega, A_1), (\omega \to \beta, A_4)\}$$

- Step 2. *Apply the bottom incision function '*$\sigma_\downarrow$*' to* $K_{A_1}{}^{\perp\!\!\perp}\beta$ *to find the set containing the least credible information objects from each* β*-kernel.*

$$\sigma_\downarrow(K_{A_1}{}^{\perp\!\!\perp}\beta) = \{(\beta, A_1), (\alpha, A_2), (\alpha \to \beta, A_2), (\alpha, A_3), (\omega, A_1)\} \cup$$
$$\cup \{(\delta \to \beta, A_1), (\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_2), (\epsilon \to \beta, A_3)\}$$

- Step 3. $K_{A_1} \ominus_{\sigma_\downarrow} \beta = K_{A_1} \setminus \sigma_\downarrow(K_{A_1}{}^{\perp\!\!\perp}\beta)$.
  $K_{A_1} \ominus_{\sigma_\downarrow} \beta = \{(\alpha \to \beta, A_4), (\omega \to \beta, A_4), (\alpha \to \delta, A_2), (\epsilon \to \beta, A_4)\}$.

Note that, in Example 12 there are kernels that differ only in the associated agent identifier. This occurs when a base has the same sentence in several information objects. Since the incision function selects the least credible information objects of each α-kernel, then as more information object containing the same belief are in K_{A_i}, more information objects will be selected by the bottom incision function. As a consequence, in some cases, this operator withdraws several information objects. In contrast, if the agent considers a compacted belief base when it applies the contraction operator, there will be less information objects selected by the incision function.

Definition 20 (Optimal contraction using plausibility) *Let* $K_{A_i} \in \mathcal{K}$, $\alpha \in \mathcal{L}$ *and let* $\sigma_\downarrow$ *be a bottom incision function for* K_{A_i}. *The operator '*$-_{\sigma_\downarrow}$*', called* optimal contraction using plausibility, *is defined as follows:*

$$K_{A_i} -_{\sigma_\downarrow} \alpha = K_{A_i} - X$$

where: $X = \{(\omega, A_j) : \omega \in Sen(\sigma_\downarrow(K^{\uparrow}_{A_i} \perp\!\!\!\perp \alpha))$ *and* $(\omega, A_j) \in K_{A_i}\}$.

Example 13 *Consider* K_{A_1} *and* '$\leq^{A_1}_{Co}$' *of Example 12. Then, suppose* A_1 *wants to contract by* β *using* '$-_{\sigma_\downarrow}$'.

- Step 1. *Find the minimal subsets that derive* β *from a compacted belief base* K_{A_1}.

$$K^{\uparrow}_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha \to \beta, A_4), (\omega, A_1), (\omega \to \beta, A_4)\} \cup$$
$$\cup \{(\alpha \to \delta, A_2), (\delta \to \beta, A_1), (\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_4)\}$$

  Note that, the policy used by the selection function (Definition 9), when we apply the compact belief base function (Definition 10), is based on a lexicographical ordering among agent identifiers. $K^{\uparrow}_{A_1} \perp\!\!\!\perp \beta = \{H_a, H_d, H_f, H_g, H_k\}$ *where*

$$H_a = \{(\beta, A_1)\} \qquad H_g = \{(\alpha, A_2), (\alpha \to \delta, A_2), (\delta \to \beta, A_1)\}$$
$$H_d = \{(\alpha, A_2), (\alpha \to \beta, A_4)\} \qquad H_k = \{(\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_4)\}$$
$$H_f = \{(\omega, A_1), (\omega \to \beta, A_4)\}$$

- Step 2. *Apply the bottom incision function* '$\sigma_\downarrow$' *to* $K^{\perp\!\!\!\perp}_{A_1}\beta$.
  $\sigma_\downarrow(K^{\perp\!\!\!\perp}_{A_1}\beta) = \{(\beta, A_1), (\alpha, A_2), (\omega, A_1), (\delta \to \beta, A_1), (\gamma, A_3), (\gamma \to \epsilon, A_2)\}$.
- Step 3. $K_{A_1} -_{\sigma_\downarrow} \beta = \{(\alpha \to \beta, A_2), (\alpha \to \beta, A_4), (\omega \to \beta, A_4), (\alpha \to \delta, A_2), (\epsilon \to \beta, A_2), (\epsilon \to \beta, A_3), (\epsilon \to \beta, A_4)\}$.

Note that, since the belief base may be non-compacted, in step 3 of Example 13, all those tuples whose beliefs were selected by the bottom incision function without regarding the respective associated agents are discarded from K_{A_1}. Besides, observe that in the last two examples the contracted belief bases have the same beliefs (see Proposition 4). However, in the latest example, the belief base contains more information objects than in the previous one (see Proposition 5). Then, this operator does not lose the associated agents of the belief remaining after the contraction. Consequently, this type of contraction is more conservative.

Proposition 4 *Let* $K_{A_i} \in \mathcal{K}$, $\alpha \in \mathcal{L}$, '$\ominus_{\sigma_\downarrow}$' *be a contraction using plausibility operator and* '$-_{\sigma_\downarrow}$' *an optimal contraction using plausibility operator, then*

$$Sen(K_{A_i} \ominus_{\sigma_\downarrow} \alpha) = Sen(K_{A_i} -_{\sigma_\downarrow} \alpha)$$

Proof: See Appendix *in Section 7.*

Proposition 5 *Let* $K_{A_i} \in \mathcal{K}$, $\alpha \in \mathcal{L}$, '$\ominus_{\sigma_\downarrow}$' *be a contraction using plausibility operator and* '$-_{\sigma_\downarrow}$' *an optimal contraction using plausibility operator, then*

$$K_{A_i} \ominus_{\sigma_\downarrow} \alpha \subseteq K_{A_i} -_{\sigma_\downarrow} \alpha$$

Proof: See Appendix *in Section 7.*

4.2.2 Properties

Next we will give the rationality postulates for optimal contraction using plausibility operator, adapting some of the postulates given in (Hansson, 1999), considering the following principle.

> **Minimal change**. As much old knowledge as possible should be retained in the revised/ contracted knowledge. That is, we should give up beliefs only when forced to do so, and then we should discard as few of them as possible.

Let A_i, A_j, $A_k, A_p \in \mathbb{A}$ and let $K_{A_i} \in \mathcal{K}$ be a belief base. The contraction operator will be represented by '$-$'. We propose the following postulates for contraction.

(CP-1) *Success:* if $\alpha \notin Cn(\emptyset)$, then $\alpha \notin Bel(K_{A_i} - \alpha)$.

The first postulate establishes that the contraction should be successful; that is, the result of contracting a belief base K_{A_i} by a sentence α (that is not a tautology) should be a new belief base that does not imply α.

(CP-2) *Inclusion:* $K_{A_i} - \alpha \subseteq K_{A_i}$.

Since $K_{A_i} - \alpha$ follows from withdrawing some beliefs from K_{A_i} without adding any belief, it is natural to think that $K_{A_i} - \alpha$ does not contain beliefs that do not belong to K_{A_i}.

(CP-3) *Uniformity:* If for all $K' \subseteq K_{A_i}$, $\alpha \in Bel(K')$ if and only if $\beta \in Bel(K')$ then $K_{A_i} - \alpha = K_{A_i} - \beta$.

This property establishes that if two sentences α and β are implied by exactly the same subsets of K_{A_i}, then the contraction of K_{A_i} by α should be equal to the contraction of K_{A_i} by β.

Next, we propose a new postulate which is an adapted version of the postulate of core-retainment defined in (Hansson, 1994): '*The beliefs that we give up in order to contract K_{A_i} by α should all be such that they contributed to the fact that K_{A_i}, but not $K_{A_i} - \alpha$, implies α. More precisely, for β to be deleted in the process of forming $K_{A_i} - \alpha$ from K_{A_i}, there should be some order in which the elements of K_{A_i} can be removed, such that the removal of β is the crucial step by which α stops to be logically implied.*' In our contraction operator, this order is based on the credibility order among agents.

(CP-4) *Minimal Plausibility Change*: If $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} - \alpha$ then there is $K' \subseteq K_{A_i}$ where $\alpha \notin Bel(K')$ but there exists $(\beta, A_j) \in K_{A_i}$ such that:

- $\alpha \in Bel(K' \cup \{(\beta, A_j)\})$,
- $p = j$ or $A_p \leq^{A_i}_{Co} A_j$, and
- for all $(\delta, A_k) \in K'$ such that $\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) \backslash \{(\delta, A_k)\})$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

The intuition behind (CP-4) is that, if β is removed, δ is preserved and both are used in a derivation of α, then β is removed because is less credible than δ. In order to remove β from K_{A_i}, we should withdraw from K_{A_i} all the information objects containing β.

Theorem 1 *Let $K_{A_i} \in \mathcal{K}$ and let '$-_{\sigma_\downarrow}$' be a contraction operator. '$-_{\sigma_\downarrow}$' is an optimal contraction using plausibility for K_{A_i} if and only if it satisfies CP-1,..., CP-4, that is, it satisfies success, inclusion, uniformity and minimal plausibility change.*
Proof: See Appendix *in Section 7.*

Note that, from CP-4 it is straightforwardly possible verify the following remark.

Remark 1 *The optimal contraction using plausibility operator follows the principle of minimal change.*

4.3 Prioritized revision using plausibility

In many multi-agent domains and applications, each agent has usually its own initial beliefs as well as knowledge acquired from other agents. In this Section and in Section 4.4, we develop two different ways in which the belief base of an agent can be rationally modified when the agent receives information from other agents that can have different degree of credibility. In the literature, there are several studied prioritized methods (e.g. *partial meet revision* (Alchourrón *et al.*, 1985) and *kernel revision* (Hansson, 1999)). In these methods, the new information has priority over the beliefs in the base of the receiver agent. Our approach is based on kernel revision and the epistemic model defined above. Thus, we focus on MSBR, where agents maintain the consistency of their belief bases.

4.3.1 Construction

The revision operator is the most complex change operator. This type of change guarantees a consistent epistemic state. When a belief base $K_{A_i} \in \mathcal{K}$ is revised by an information object (α, A_j) we will have two tasks:

- to maintain the consistency of K_{A_i}. If α is inconsistent with $Bel(K_{A_i})$, that is $\neg\alpha \in Bel(K_{A_i})$, a deeper analysis is required because it is necessary to erase some information objects from K_{A_i}.

- to add (α, A_j) to K_{A_i}. This is the most simple task to characterize from the logical point of view because it consists only in the addition of new information object. As showed above, if $\alpha \in Bel(K_{A_i})$ then this operation could increase the plausibility of α.

The first task can be accomplished contracting by $\neg\alpha$. The second task can be accomplished expanding by (α, A_j). If a belief base does not imply $\neg\alpha$, then (α, A_j) can be added without loss of consistency. This composition is based on the *Levi identity* (Gärdenfors, 1981; Alchourrón *et al.*, 1985), which proposes that a revision can be constructed out of two sub-operations: a contraction by $\neg\alpha$ and an expansion by (α, A_j).

Definition 21 (Prioritized revision using plausibility) *Let $K_{A_i} \in \mathcal{K}$, let (α, A_j) be an information object and let $\sigma_\downarrow$ a bottom incision function for K_{A_i}. Let $-_{\sigma_\downarrow}$ be the optimal contraction using plausibility operator and $+$ the expansion using plausibility operator. The operator '$*_{\sigma_\downarrow}$', called prioritized revision using plausibility, is defined as follows:*

$$K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) = (K_{A_i} -_{\sigma_\downarrow} \neg\alpha) + (\alpha, A_j)$$

Example 14 *Consider the set $\mathbb{A} = \{A_1, A_2, A_3, A_4, A_5\}$ where the credibility order according to A_1 is $A_1 <^{A_1}_{Co} A_2 =^{A_1}_{Co} A_3 <^{A_1}_{Co} A_4 <^{A_1}_{Co} A_5$. Suppose that the agent A_1 has the following belief base $K_{A_1} = \{(\beta, A_1), (\alpha, A_2), (\alpha, A_3), (\alpha \to \beta, A_2), (\alpha \to \beta, A_4), (\omega, A_1), (\omega \to \beta, A_4), (\alpha \to \delta, A_2), (\delta \to \beta, A_1), (\gamma, A_3), (\gamma \to \epsilon, A_2), (\epsilon \to \beta, A_2), (\epsilon \to \beta, A_3), (\epsilon \to \beta, A_4)\}$. Then, suppose A_1 wants to revise by $(\neg\beta, A_5)$ using '$*_{\sigma_\downarrow}$'. Since $\beta \in Bel(K_{A_1})$ then it is necessary to contract K_{A_1} by β and then expand K_{A_1} by $(\neg\beta, A_5)$. Thus, $K_{A_1} *_{\sigma_\downarrow} (\neg\beta, A_5) = \{(\alpha \to \beta, A_2), (\alpha \to \beta, A_4), (\omega \to \beta, A_4), (\alpha \to \delta, A_2), (\epsilon \to \beta, A_2), (\epsilon \to \beta, A_3), (\epsilon \to \beta, A_4)\} \cup \{(\neg\beta, A_5)\}$.*

4.3.2 Properties

Next we will give the rationality postulates for prioritized revision using plausibility operator. We must introduce the following principle, similar to the principle proposed in (Dalal, 1988).

> **Maintenance of consistency**. If a belief base K and a belief α are both consistent, then K revised by α is consistent.

Let A_i, A_j, A_k, A_p, $A_q \in \mathbb{A}$ and let $K_{A_i} \in \mathcal{K}$ be a belief base. The prioritized revision operator will be represented by '$*$'.

(RP-1) *Success:* $(\alpha, A_j) \in K_{A_i} * (\alpha, A_j)$.

Since the revision operator defined here is considered prioritized; that is, the new information has priority, the first postulate we give establishes that the revision should be successful. That is, the result of revising a belief base K_{A_i} by an information object (α, A_j) should be a new belief base that contains (α, A_j).

(RP-2) *Inclusion:* $K_{A_i} * (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$.

A particular case in the revision process occurs when a belief base K_{A_i} is revised by (α, A_j) and $\neg\alpha \in Bel(K_{A_i})$. In this case, before adding (α, A_j), $\neg\alpha$ should be withdrawn from K_{A_i}. Hence, if $\neg\alpha \in Bel(K_{A_i})$ then the revision of K_{A_i} by (α, A_j) is contained in the expansion of K_{A_i} by (α, A_j). In contrast, if $\alpha \in Bel(K_{A_i})$ (*i.e.*, α is consistent with K_{A_i}) then the revision operation is equivalent to an expansion operation.

(RP-3) *Consistency*: if α is consistent then $K_{A_i} * (\alpha, A_j)$ is consistent.

The main aim of the revision operator is to hold consistency in the belief base revised. However, there exist special cases in which this is not possible. If a belief base is revised by an information object containing a contradictory sentence, then the resultant belief base is inconsistent. Hence, the revision operator should preserve the consistency in the belief base if and only if an information object containing a contradictory sentence is not being added to the belief base.

(RP-4) *Uniformity:* If for all $K' \subseteq K_{A_i}$, $\{\alpha\} \cup Sen(K') \vdash \perp$ if and only if $\{\beta\} \cup Sen(K') \vdash \perp$ then $K_{A_i} \cap (K_{A_i} * (\alpha, A_j)) = K_{A_i} \cap (K_{A_i} * (\beta, A_k))$.

This postulate determines that if two beliefs α and β are inconsistent with the same sub-bases of K_{A_i} then K_{A_i} revised by information objects containing those beliefs should preserve the same information objects from K_{A_i}.

(RP-5) *Minimal Plausibility Change*: If $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} * (\alpha, A_k)$ then there is $K' \subseteq K_{A_i}$ where $\neg\alpha \notin Bel(K')$ but there exists $(\beta, A_j) \in K_{A_i}$ such that:

- $\neg\alpha \in Bel(K' \cup \{(\beta, A_j)\})$,
- $p = j$ or $A_p \leq_{Co}^{A_i} A_j$, and
- for all $(\delta, A_q) \in K'$ such that $\neg\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) \backslash \{(\delta, A_q)\})$ it holds that $A_j \leq_{Co}^{A_i} A_q$.

The intuition behind this postulate is similar to that of the *minimal plausibility change* postulate (CP-4) for contractions introduced above.

Theorem 2 *Let $K_{A_i} \in \mathcal{K}$ and let '$*_{\sigma_\downarrow}$' be a revision operator. '$*_{\sigma_\downarrow}$' is a* prioritized revision using plausibility *for K_{A_i} if and only if it satisfies RP-1, ..., RP-5, that is, it satisfies* success, inclusion, consistency, uniformity *and* minimal plausibility change.
Proof: See Appendix *in Section 7.*

Note that, from RP-3 and RP-5, it is straightforwardly possible verify the following remark.

Remark 2 *The prioritized revision using plausibility operator follows the principles of minimal change and maintenance of consistency.*

Proposition 6 *If '+' satisfies EP-1,...,EP-5 and '$-_{\sigma_\downarrow}$' satisfies CP-1,...,CP-4 then '$*_{\sigma_\downarrow}$' satisfies RP-1,...,RP-5.*

Proof: See Appendix *in Section 7.*

4.4 Non prioritized revision using plausibility

A prioritized revision operator is characterized by success postulate from which we may infer that $\alpha \in Bel(K_{A_i} * (\alpha, A_j))$. That is, the incoming information has priority over the beliefs in the base of the receiver agent. However, as is mentioned in Fermé and Hansson (1999), this is an unrealistic feature, since actual epistemic agents, when confronted with information that contradicts previous beliefs, often reject it. Several models of BR have been developed that allow for two options: either the new information is completely accepted or it is completely rejected (Hansson, 1997; Makinson, 1997). Below, a non-prioritized revision operator for our proposed epistemic model is introduced.

When an agent always acquires information from the same source, then a prioritized method can be used. Nevertheless, if information comes from different sources, and this sources are not equally credible, a non-prioritized method can be more adequate. This occurs in many multi-agent domains and applications. Thus, we focus on a non-prioritized BR operator that is based on the credibility ordering among agents. We propose a method for analyzing the information received; if inconsistency arises, the credibility order is used to decide which information prevails. Thus, we show that, with this new revision operator, incoming information cannot be accepted when the receiver agent has more credible beliefs that contradict the new information.

4.4.1 Construction

When a belief base $K \in \mathcal{K}$ is revised by an information object $I = (\alpha, A_i)$ using a non-prioritized revision operator there are two cases:

- α is consistent with $Bel(K)$. In this case, the operator is equivalent to the prioritized version.
- α is inconsistent with $Bel(K)$, that is $\neg\alpha \in Bel(K)$. First, it is necessary to determine whether the sentence will be accepted; and then if the input is accepted, then the operator is equivalent to the prioritized version.

According to this, two options arise: completely accept all the input, or completely reject all the input. In the literature there are other operators which may partially accept the new information, for instance, *Revision by a Set of Sentences* defined on belief bases (Falappa *et al.*, 2002) and *Selective Revision* defined on belief sets (Farmé and Hansson, 1999).

Definition 22 (Non-prioritized revision using plausibility) *Let K_{A_i} be a belief base in $\mathcal{K}$, (α, A_j) be an information object, and $\sigma_\downarrow$ a bottom incision function for K_{A_i}. Let $*_{\sigma_\downarrow}$ be the prioritized revision using plausibility operator and + be the expansion using plausibility operator. The operator '$\circ_{\sigma_\downarrow}$', called non-prioritized revision using plausibility, is defined as follows:*

$$K_{A_i} \circ_{\sigma_\downarrow} (\alpha, A_j) = \begin{cases} K_{A_i} + (\alpha, A_j) & \text{if } \neg\alpha \notin Bel(K_{A_i}) \\ K_{A_i} & \text{if } \neg\alpha \in Bel(K_{A_i}) \text{ and } A_j <^{A_i}_{Co} Pl(\neg\alpha, K_{A_i}) \\ K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) & \text{if } \neg\alpha \in Bel(K_{A_i}) \text{ and } Pl(\neg\alpha, K_{A_i}) \leq^{A_i}_{Co} A_j \end{cases}$$

Note that, if the incoming information is as credible as the beliefs which are possibly withdrawn, this operator prioritizes the input. That is, if an agent receives information from the same informant, it is natural that the more recent information will be accepted.

Example 15 *Consider K_{A_1} and '$\leq^{A_1}_{Co}$' of Example 14. Then, suppose A_1 wants to revise by $(\neg\beta, A_5)$ using '$\circ_{\sigma_\downarrow}$'. Since $Pl(\beta, K_{A_1}) = A_2 <^{A_1}_{Co} A_5$ then $K_{A_1} \circ_{\sigma_\downarrow} (\neg\beta, A_5) = K_{A_1} *_{\sigma_\downarrow} (\neg\beta, A_5) = \{(\alpha \rightarrow \beta, A_2), (\alpha \rightarrow \beta, A_4), (\omega \rightarrow \beta, A_4), (\alpha \rightarrow \delta, A_2), (\epsilon \rightarrow \beta, A_2), (\epsilon \rightarrow \beta, A_3), (\epsilon \rightarrow \beta, A_4)\} \cup \{(\neg\beta, A_5)\}$.*

It is important to note that if the input of Example 14 is $(\neg\beta, A_1)$ rather than $(\neg\beta, A_5)$, then the revision will not have effect because $A_1 <^{A_1}_{Co} A_2$. Thus, this operator will never discard more plausible sentences than the input.

4.4.2 Properties: success postulate must be weakened

An operator defined following the Definition 22, in general, satisfies the same postulates that satisfies the prioritized version. However, we must introduce the following principle.

> **Non-prioritization principle**. If a belief base is revised by an information object (α, A_j), then α will not be necessarily accepted in the revised belief base. A sentence α will be accepted in the revised belief base only when its informant A_j is sufficiently plausible or credible.

Let A_i, A_j and $A_k \in \mathbb{A}$, let $K_{A_i} \in \mathcal{K}$ be a belief base and let '$*$' be a prioritized revision operator on K_{A_i}. The non-prioritized revision operator will be represented by '$\circ$'. In the general case, the non-prioritized revision operator will be equal to a prioritized revision operator. However, in some particular cases, '$\circ$' does not satisfies *success* and, therefore, we need weaker versions of this postulate.

(NRP-1) *Weak Success:* if $\neg\alpha \notin Bel(K)$ then $(\alpha, A_j) \in K_{A_i} \circ (\alpha, A_j)$.

This postulate establishes that α is accepted in the revised belief base if $\neg\alpha$ is not derived in the original belief base.

(NRP-2) *Relative Success:* $K_{A_i} \circ (\alpha, A_j) = K_{A_i}$ or $(\alpha, A_j) \in K_{A_i} \circ (\alpha, A_j)$.

This postulate, inspired in (Hansson *et al.*, 2001), says that all or nothing is accepted. That is, α is accepted in the revised belief base or nothing changes.

Both *weak success* and *relative success* do not capture the intuitions behind the non-prioritization principle. Therefore, we propose the following postulate, called *Conditional Success*.

(NRP-3) *Conditional Success*: If $(\beta, A_k) \in K^{\uparrow}_{A_i}$ and $\beta \notin Sen(K_{A_i} * (\alpha, A_j))$ then $(\alpha, A_j) \in K_{A_i} \circ (\alpha, A_j)$ if and only if $A_k \leq^{A_i}_{Co} A_j$.

This postulate establishes that α is accepted in the revised belief base when its informant is sufficiently plausible.

Following Definition 22 it is possible to show that let $K_{A_i} \in \mathcal{K}$, '$\circ_{\sigma_\downarrow}$' is a *non-prioritized revision using plausibility* for K_{A_i} if and only if it satisfies *uniformity*, *consistency*, *conditional success*, *inclusion* and *minimal plausibility change*. Hence, we can straightforwardly note that the non-prioritized revision using plausibility operator follows the principles of minimal change, maintenance of consistency and non-prioritization.

4.5 Application example

Consider the following scenario. An agent A_1 wants to travel to a village in a mountain and knows from the Tourist Information Office (A_t) that if it rains (ρ), then the road to the village is not open ($\neg o$). Hence, $K_{A_1} = \{(\rho \rightarrow \neg o, A_t)\}$. Agent A_1 also knows that it can obtain information from other sources: an agent A_c coming down from the village, an agent A_g at the gas station, an agent A_r at some restaurant, or the weather report on the radio (A_w). The credibility order of A_1 is $A_1 <^{A_1}_{Co} A_r <^{A_1}_{Co} A_g <^{A_1}_{Co} A_t <^{A_1}_{Co} A_w <^{A_1}_{Co} A_c$ and A_1 uses the non-prioritized operator $\circ_{\sigma_\downarrow}$ introduced above to revise its beliefs.

Then, the agent A_1 obtains from A_g the information object $I_1 = (\rho, A_g)$ and revises its belief base: $K_{A_1} \circ_{\sigma_\downarrow} (\rho, A_g) = \{(\rho \rightarrow \neg o, A_t), (\rho, A_g)\}$. Observe that I_1 is added to its belief base, and now $\neg o \in Bel(K_{A_1})$ (i.e. with this new information A_1 believes that the road is not open). Later, A_1 obtains from A_r the information object $I_2 = (\neg\rho, A_r)$. Since $\rho \in Bel(K_{A_1})$ and $A_r <^{A_1}_{Co} Pl(\rho, K_{A_1}) = A_g$, then I_2 is rejected and its belief base does not change. Agent A_1 then obtains $I_3 = (\rho, A_w)$ from the weather report and revises K_{A_1} by I_3. Since I_3 is not contradictory with K_{A_1}, I_3 is added: $K_{A_1} \circ_{\sigma_\downarrow} (\rho, A_w) = \{(\rho \rightarrow \neg o, A_t), (\rho, A_g), (\rho, A_w)\}$. Observe that the plausibility of ρ and $\neg o$ are both increased.

Finally, A_1 obtains $I_4 = (o, A_c)$ (the road is open) from an agent A_c that is coming down from the village. Since this new information is contradictory with A_1 beliefs (because $\neg o \in Bel(K_{A_1})$) then the kernel set for $\neg o$ is obtained: $K^{\uparrow}_{A_1} \perp\!\!\!\perp \neg o = \{\{(\rho \rightarrow \neg o, A_t), (\rho, A_w)\}\}$. Then, $(\rho \rightarrow \neg o, A_t)$ is selected to withdraw it, and hence, $K_{A_1} \circ_{\sigma_\downarrow} (o, A_c) = \{(o, A_c), (\rho, A_g), (\rho, A_w)\}$. Observe that now $o \in Bel(K_{A_1})$. That is, since the information that *the road is open* is more credible than the most plausible derivation for $\neg o$ (*the road is not open*), then the revision operator contracts K_{A_1} using the incision function which selects $(\rho \rightarrow \neg o)$. Then I_4 is added to the agent belief base in a consistent way. Thus, A_1 finally believes that the road is open. Note that the sentences selected by the incision function in the revision are less credible than o.

5 Forwarding Information

In the previous sections we have introduced a formalism for MSBR. Using that formalism, agents can acquire information objects from multiple sources and incorporate them into their proper beliefs. Both prioritized and non-prioritized revision operators were introduced using the credibility order of each agent in order to decide which information prevails. Nevertheless, nothing was said about how an agent can forward information that is obtained from others. In the following, we assume that all agents use the epistemic model introduced above, have their own credibility order, and incorporate information objects through some of the revision operators defined above.

Although the contribution of this paper is focused on the formalism presented above, in this section we briefly comment different strategies for forwarding information to other agents. In particular, we study how to rationally choose meta-information to be sent as a label in the information objects. The choice of the agent identifier to be sent with the piece of information is crucial as it influences the decision of the receiver about whether to accept the transmitted information. Thus, it is in the interest of the sending agent, and in fact in the interest of the whole coalition of agents, to choose this meta-information carefully.

As stated above, when an agent sends information to another agent, it sends information objects. Consider the set of agent identifiers $\mathbb{A} = \{A_1, A_2, A_3, A_4\}$ where $A_1 <^{A_1}_{Co} A_2 =^{A_1}_{Co} A_3 <^{A_1}_{Co} A_4$. Suppose that the belief base of the agent A_1 is $K_{A_1} = \{(\alpha, A_2), (\alpha, A_4), (\beta, A_3), (\beta \rightarrow \alpha, A_1)\}$. If A_1 wants to send α to A_2, it should send a tuple $I = (\alpha, Agent)$, and it is clear that there are several choices for the identifier '*Agent*' of I.

In Krümpelmann *et al.* (2009), we describe different criteria for forwarding information that determine which agent identifier is considered by the receiver at the moment of reasoning. That is to say, we analyze different alternatives which determine which agent identifier is sent in the information object. Some of those are: '*Sender identifier criterion*' which, as in Dragoni *et al.* (1994), suggests sending the proper sender (A_1) in the information object; '*Source identifier criterion*' that proposes sending one of the identifiers stored with α in the sender's base (e.g. A_2, A_4) that can be one of them arbitrarily or the more credible of them as is suggested by the '*Combined criterion*'. In Krümpelmann *et al.* (2009) is shown that there are some examples in which these simple criteria can select an unappropriated identifier.

In this section, we show a more elaborated criterion that takes the plausibility of sentences obtained from agents credibility into consideration. This criterion calculates the plausibility of a sentence α based on all its proofs before being forwarded. This calculation should return an agent identifier which will be used as the agent identifier of α. Thus, a forwarding criterion can be implemented by sending an information object $I = (\alpha, A_i)$ where A_i is the agent identifier obtained using the plausibility function defined above; that is, $A_i = Pl(\alpha, K_{A_1})$ where A_1 is the forwarder. For instance, if the agentA_1 of application example of Section 4.5 wants to send the sentence ρ, it will send the information object (ρ, A_w).

Example 16 *Let us consider Example 9 again. If the agent A_1 wishes to send β to agent A_4 then, according to the plausibility based criterion, A_1 will send the information object $(\beta, Pl(\beta, K_{A_1}))$ to A_4. That is, A_1 sends, based on its belief base K_{A_1} and its credibility order '$\leq_{co}^{A_1}$', (β, A_2) to A_4.*

An important decision we made is to forward an agent identifier with a sentence rather than a credibility label in order to give additional information to the beliefs. One reason for this decision is that we achieve a more dynamic framework since the evaluation of the credibility of the agent identifiers is separated by use of the assessment function. Note that the assessment function may change in time realizing dynamic assessments. Hence, the credibility order among agents can be changed without changing the knowledge base or the operator. That is, if the credibility order among agents changes, then the plausibility of all sentences will also change without having to modify the belief base of the agent. Another reason for this decision is that since each agent has its own *assessment* (as stated in Section 2), it is more suitable to send agent identifiers and then the receiver agent can evaluate the received belief based on the credibility it has according to its own assessment. This means that the sending agent expresses that it considers the information it transmits as credible as it considers the agent identifier in the information object. Now it is up to the receiver to assess how credible it considers each agent from its perspective using its own assessment function. We believe that this represents an advanced way of communication in MAS.

5 Conclusions and Related Works

In this work, we have proposed a general framework to deal with the knowledge dynamics of a MAS. We have introduced an epistemic model and a set of operators to change the belief base of each agent: expansion, contraction, prioritized revision and non-prioritized revision. We have defined a set of postulates for every operator, we have proved representation theorems for the more important changes (contractions and revisions), and we have shown some interesting principles for each of them. We have shown how these operators can be used for MSBR system weighing beliefs following a credibility order among agents, giving illustrative examples, and showing that these operators may improve the collective reasoning of a MAS.

In the literature, different formalisms have been presented to deal with MABR (Malheiro *et al.*, 1994; Kfir-Dahav and Tennenholz, 1996; Liu and Williams, 1999, 2001) where the overall BR of agent teams is investigated. In contrast to these, we focused on MSBR which is one of the essential components of MABR. Here, the agents maintain the consistency of their belief bases. Two other approaches that cope with MSBR are Dragoni *et al.* (1994) and Cantwell (1998). The epistemic model in these works is similar to the one we defined in Section 2; however, our theory change is different to theirs. That is, like us, both consider that the reliability of the source affects the credibility of incoming information, and this reliability is used for making decisions. Nevertheless, these two approaches differ from ours in several issues as is detailed below.

In Dragoni *et al.* (1994, 1997), it is considered that agents detect and store in tables the *nogoods*, which are the minimally inconsistent subsets of their knowledge bases. A *good* is a subset of the knowledge base such that: it is not inconsistent (it is not a superset of a *nogood*), and if augmented with whatever else assumption in knowledge base, it becomes inconsistent. In contrast to our approach, they do not remove beliefs to avoid a contradiction, but quite more generally, they choose which is the new preferred *good* among them in knowledge base. In our model, we obtain the kernel sets to cut some sentences, thus we break the contradictions if it is necessary.

Like us, they propose to store additional information with each sentence. However, their tuples contain five elements: <Identifier, Sentence, Origin Set (OS), Source, Credibility>, where OS records the assumption nodes upon which it really ultimately depends (as derived by the theorem prover). In contrast to them, in our model a tuple only store a sentence and an associated agent, but a tuple does not store the credibility. That is, in our model, the plausibility of a sentence is not explicitly stored with it, as it is in Dragoni *et al.* (1994). Thus, when the plausibility of some sentence is needed, the *plausibility function* should be applied. As is shown in Example 17, given a sentence α, its plausibility depends on its proofs (α-*kernels*). Therefore, if one of the sentences of these proof changes, then the plausibility of α may change. Hence, if the credibility order is replaced, then the sentence plausibility may change without changing the belief base.

Example 17 *Consider a set* $\mathbb{A} = \{A_1, A_2\}$ *where the credibility order is* $A_1 \leq_{Co}^{A_1} A_2, K_{A_1} = \{(\alpha, A_1), (\alpha \rightarrow \beta, A_2)\}$ *and* $K_{A_2} = \{(\alpha, A_2)\}$. *By Definition 14,* $Pl(\beta, K_{A_1}) = A_1$. *Now, suppose that* A_1 *receives from* A_2 *the belief* α. *Now* $K_{A_1} = \{(\alpha, A_1), (\alpha, A_2), (\alpha \rightarrow \beta, A_2)\}$ *and* A_1 *has two derivations for* β, *hence* $Pl(\beta, K_{A_1}) = A_2$. *Observe that plausibility of* β *is increased.*

The communication policy in Dragoni *et al.* (1994) is that agents do not communicate the sources of the assumptions, but they present themselves as completely responsible for the knowledge they are passing on; receiving agents consider the sending ones as the sources of all the assumptions they are receiving from them. In Section 5, we showed a more elaborated criterion which proposes to calculate the plausibility of a sentence based on all its proofs before being forwarded. An important decision we made was to forward an agent identifier with a sentence rather than a credibility label in order to give additional information to the beliefs. One reason for this decision is that since each agent has its own *assessment*, it is more suitable to send agent identifiers so in this way, the receiver agent can evaluate the belief received based on the credibility it has according to its own assessment. Another reason is that we achieve a more dynamic framework since the evaluation of the credibility of the agent identifiers is separated by use of the assessment function.

In Cantwell (1998), a *scenario* (set of incoming information) presented by a source is treated as a whole and not sentence by sentence, and therefore, it can be inconsistent. A relation of *trustworthiness* is introduced over sets of sources and not between single sources. Besides, if two sources give the same piece of information α, and a single agent gives $\neg\alpha$, then α will be preferred, that is, the decision is based on majority. In his approach, the order in which the evidence is considered does not seem to be important. However, in our work, the order in which beliefs are considered is important: If an agent receives α and then receives $\neg\alpha$ and both have the same plausibility, then $\neg\alpha$ will be rejected.

Our work has some link with the idea of epistemic entrenchment (Gärdenfors and Makinson, 1988; Rott, 1992). Here the sentence plausibility is used in a similar way to epistemic entrenchment to modify knowledge. However, there are some differences between them. For instance, in our work the order among sentences is based on the informants, whereas in Gärdenfors and Makinson (1988) the order among sentences is implicitly defined over belief states represented by belief sets.

When we count on a MAS that has only one agent, the new operator is very drastic. In this scenario there is no order among agents. The same happens when all the agents of a MAS have equal credibility. In these cases the bottom incision function does not have enough information to select sentences and it will erase all sentences in the α-kernels. This behavior is similar to *full meet revision* on belief bases (Hansson, 1999). Nevertheless, when a MAS has several agents with different credibility and it is necessary to represent knowledge dynamics of the agents, plausibility seems to be a good criteria.

In Benferhat *et al.* (1993), several methods to deal with inconsistency are investigated by defining notions of consequence capable of inferring non-trivial conclusions from an inconsistent knowledge base. It is clear that the methods proposed here and in Benferhat *et al.* (1993) follow different attitudes when facing inconsistent knowledge. In (Benferhat *et al.*, 1993) inconsistency-tolerant consequence relations in layered knowledge bases are proposed, whereas here a revision operator is defined.

It is important to note that the revision operator proposed here is similar to the revision operator proposed in Benferhat *et al.* (2002). However, these operators are built in a different way. In Benferhat *et al.* (2002), the epistemic state is represented by a possibility distribution which is a mapping from the set of classical interpretations or worlds to the [0,1] interval. This distribution represents the degree of compatibility of the interpretations with the available information and the revision is done over the possibility distribution. This revision modifies the ranking of interpretations so as to give priority to the input information. The input must be incorporated in the epistemic state; in other words, it takes priority over information in the epistemic state. They discuss the revision with respect to uncertain information; the input is of the form (ϕ, a), which means that the classical formula ϕ should be believed to a degree of certainty of exactly a.

Both approaches differ in some interesting ways. A first difference occurs in the way they handle the epistemic state. In Benferhat *et al.* (2002), the authors use belief sets, whereas we use belief bases. The use of belief bases makes the representation of the agent's cognitive state more natural and computationally tractable. That is, following Hansson (1999, page 24), we consider that agents' beliefs could be represented by a limited number of sentences that correspond to the explicit beliefs of the agent. Another important difference, related to the intention of using the operator in a MAS environment, is the additional information added to each belief. Here, to decide whether to reject or accept a new belief, a comparison criterion among beliefs is defined. This criterion (called plausibility) is based on the credibility order among agents. We have assumed that this order is fixed; however, this order can be changed without affecting the definition of the operator. This characteristic is one of the motivations for using agent identifiers instead of representing the plausibility of a sentence as in Benferhat *et al.* (2002). Moreover, here a total order among agents is necessary, but this assumption can be relaxed considering a partial order among agents.

As future work, we will try to extend the applications of the proposed framework in environments in which the credibility of agents changes, and therefore, the plausibility of beliefs (and the results of changes) can be dynamically modified. That is, since the revision process is based on the credibility order among agents, it is possible to define an operator to revise the credibility order. This will allow to represent changes over the credibility order.

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Appendix A

Proposition 3. Let $K_{A_i} \in \mathcal{K}$ and let $\alpha \in Bel(K_{A_i})$, then the plausibility of α in the belief base K_{A_i} is equal to the plausibility of α in the compacted belief base $K^{\uparrow}_{A_i}$. That is,
$Pl(\alpha, K_{A_i}) = \mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K^{\uparrow\perp\!\!\perp}_{A_i}\alpha} min(X)))) = \mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K^{\perp\!\!\perp}_{A_i}\alpha} min(X))))$.

Proof

Let $\mathbb{A} = \{A_1, \ldots, A_n\}$. If $K^{\uparrow}_{A_i} = K_{A_i}$ then is trivially proved. If $K^{\uparrow}_{A_i} \subset K_{A_i}$ then there exists some sentence β in $Sen(K_{A_i})$ such that β occurs in m tuples in K_{A_i} $(m \geq 2)$. Consider $(\beta, A_i) \in X$ $(1 \leq i \leq n)$ for some $X \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha$ then $K^{\perp\!\!\perp}_{A_i}\alpha$ will have m β*−kernels* $(X, Y_1, \ldots, Y_{m-1})$ such that they will differ only in the tuple containing β. Suppose that $(\beta, A^p_j) \in Y_p$ for all p $(1 \leq p \leq m-1, j \neq i$ and $1 \leq j \leq n)$. Next, we will prove that X will contain the only relevant tuples to compute the plausibility of α. There are three cases:

- If $min(X) = (\beta, A_i)$, then we have that $min(Y_p) = (\beta, A^p_j)$. By Definition 10, $A^p_j \leq^{A_i}_{Co} A_i$ for all p. Moreover $X, Y_1, \ldots, Y_{m-1}$ differ only in the tuple in that is β. Therefore, $max((\beta, A_i),(\beta, A^1_j),\ldots,(\beta, A^{m-1}_j)) = (\beta, A_i) \in X$. In case that $A^p_j =^{A_i}_{Co} A_i$, note that the selection function follows the same policy either in compact belief base function as plausibility function. Hence, the selection function returns the same agent identifier in both cases.
- If $min(X) \neq (\beta, A_i)$ and $min(Y_p) \neq (\beta, A^p_j)$, then min will return the same tuple in all the cases. Then $X, Y_1, \ldots, Y_{m-1}$ differ only in the tuple containing β.
- If $min(X) \neq (\beta, A_i)$ (suppose that $min(X) = (\omega, A_j)$) and $min(Y_p) = (\beta, A^p_j)$ for some p, then since $(\omega, A_j) \in Y_p$, $A^p_j \leq^{A_i}_{Co} A_j$. Note that, if $min(Y_p) \neq (\beta, A^p_j)$ then by the previous case $min(Y_p) = (\omega, A_j)$. Hence, $max(min(X) \cup min(Y_1) \cup \ldots \cup min(Y_{m-1})) = (\omega, A_j) \in X$. In case that $A^p_j =^{A_i}_{Co} A_i$, note that the selection function follows the same policy either in compact belief base function as plausibility function. Hence, the selection function returns the same agent identifier in both cases.

Therefore, from the m β-kernels $(X, Y_1, \ldots, Y_{m-1})$ only X will contain the relevant tuples to calculate the plausibility of α. Then, $Pl(\alpha, K_{A_i})$ is equal to:

$$\mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K^{\uparrow\perp\!\!\perp}_{A_i}\alpha} min(X)))) = \mathcal{S}_{A_i}(Ag(max(\bigcup_{X \in K^{\perp\!\!\perp}_{A_i}\alpha} min(X))))$$

Proposition 4. Let $K_{A_i} \in \mathcal{K}$, $\alpha \in \mathcal{L}$, '$\ominus_{\sigma_\downarrow}$' be a contraction using plausibility operator and '$-_{\sigma_\downarrow}$' an optimal contraction using plausibility operator, then

$$Sen(K_{A_i} \ominus_{\sigma_\downarrow} \alpha) = Sen(K_{A_i} -_{\sigma_\downarrow} \alpha)$$

Proof

($\subseteq$) Let $\beta \in Sen(K_{A_i} \ominus_{\sigma_\downarrow} \alpha)$. We should prove that $\beta \in Sen(K_{A_i} -_{\sigma_\downarrow} \alpha)$. Then, by Definition 3 there exists an information object $(\beta, A_j) \in K_{A_i} \ominus_{\sigma_\downarrow} \alpha$. It follows from Definition 19 that $(\beta, A_j) \in (K_{A_i} \backslash \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha))$. Thus $(\beta, A_j) \in K_{A_i}$ and $(\beta, A_j) \notin \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Since by Proposition 1 $K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha \subseteq K^{\perp\!\!\perp}_{A_i}\alpha$ then, by Definition 18, $\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha) \subseteq \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$ and $(\beta, A_j) \notin \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)$. Hence, by Definition 3, $\beta \notin Sen(\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha))$ and, by Definition 20, $\beta \in Sen(K_{A_i} -_{\sigma_\downarrow} \alpha)$.

($\supseteq$) Let $\beta \in Sen(K_{A_i} -_{\sigma_\downarrow} \alpha)$. We should prove that $\beta \in Sen(K_{A_i} \ominus_{\sigma_\downarrow} \alpha)$. Then, by Definition 3, there exists an information object $(\beta, A_j) \in K_{A_i} -_{\sigma_\downarrow} \alpha$. It follows from Definition 20 that $(\beta, A_j) \in K_{A_i} \backslash X$ where $X = \{(\omega, A_k) : \omega \in Sen(\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)) \text{and} (\omega, \mathrm{A_k}) \in \mathrm{K_{A_i}}\}$. Thus $(\beta, A_j) \in K_{A_i}$ and $(\beta, A_j) \notin X$. Then there exists $(\beta, A_p) \in K^{\uparrow}_{A_i}$ such that $(\beta, A_p) \notin \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)$ with $A_j \leq^{A_i}_{Co} A_p$. In case that $(\beta, A_p) \notin \bigcup(K^{\perp\!\!\perp}_{A_i}\alpha)$ then, by Definition 18, $(\beta, A_p) \notin \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Since $(\beta, A_p) \notin \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)$, it follows from Definition 18 that there exists $(\delta, A_q) \in Y \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha$ such that $A_q \leq^{A_i}_{Co} A_p$. Since $(\delta, A_q) \in K^{\uparrow}_{A_i}$ (and, by Proposition 1, $(\delta, A_q) \in K_{A_i}$) then for all $Z \in K^{\perp\!\!\perp}_{A_i}\alpha$ such that $Sen(Y) = Sen(Z)$, $(\beta, A_p) \notin Z \cap \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Thus $(\beta, A_p) \notin \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Then $(\beta, A_p) \in K_{A_i} \backslash \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Hence, by Definition 19, $\beta \in Sen(K_{A_i} \ominus_{\sigma_\downarrow} \alpha)$.

Proposition 5. Let $K_{A_i} \in \mathcal{K}$, $\alpha \in \mathcal{L}$, '$\ominus_{\sigma_\downarrow}$' be a contraction using plausibility operator and '$-_{\sigma_\downarrow}$' an optimal contraction using plausibility operator, then

$$K_{A_i} \ominus_{\sigma_\downarrow} \alpha \subseteq K_{A_i} -_{\sigma_\downarrow} \alpha$$

Proof Let $(\beta, A_j) \in K_{A_i} \ominus_{\sigma_\downarrow} \alpha$, we should prove that $(\beta, A_j) \in K_{A_i} -_{\sigma_\downarrow} \alpha$. It follows from Definition 19 that $(\beta, A_j) \in (K_{A_i} - \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha))$. Thus $(\beta, A_j) \in K_{A_i}$ and $(\beta, A_j) \notin \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Since by Proposition 1 $K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha \subseteq K^{\perp\!\!\perp}_{A_i}\alpha$, then by Definition 18 $\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha) \subseteq \sigma_\downarrow(K^{\perp\!\!\perp}_{A_i}\alpha)$. Thus $(\beta, A_j) \notin \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)$. Hence, by Definition 3, $\beta \notin Sen(\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha))$. Then, by Definition 20, $(\beta, A_j) \in K_{A_i} -_{\sigma_\downarrow} \alpha$.

Next, we give a lemma used in the representation theorem of contraction operator (Theorem 1). Note that this Lemma is an adapted version of a property defined in Hansson (1999).

Lemma 1. $K^{\uparrow\perp\!\!\perp}\alpha = K^{\uparrow\perp\!\!\perp}\beta$ if and only if for all subsets K' of $K^{\uparrow}$: $\alpha \in Bel(K')$ if and only if $\beta \in Bel(K')$.

Proof We will use *reductio by absurdum*.
($\Rightarrow$) Suppose that there is some subset B of $K^{\uparrow}$ such that $\alpha \in Bel(B)$ and $\beta \notin Bel(B)$. By compactness, there is some subset B' of $K^{\uparrow}$ such that $\alpha \in Bel(B')$. Then, there is some element B'' of $K^{\uparrow\perp\!\!\perp}\alpha$ such that $B'' \subseteq B'$. Since $B'' \subseteq B$ and $\beta \notin Bel(B)$, we have $\beta \notin Bel(B'')$, so that $B'' \notin K^{\uparrow\perp\!\!\perp}\beta$. Then $B'' \in K^{\uparrow\perp\!\!\perp}\alpha$ and $B'' \notin K^{\uparrow\perp\!\!\perp}\beta$ contrary to $K^{\uparrow\perp\!\!\perp}\alpha = K^{\uparrow\perp\!\!\perp}\beta$.
($\Leftarrow$) Suppose that $K^{\uparrow\perp\!\!\perp}\alpha \neq K^{\uparrow\perp\!\!\perp}\beta$. We may assume that there is some $X \in K^{\uparrow\perp\!\!\perp}\alpha$ such that $X \notin K^{\uparrow\perp\!\!\perp}\beta$. There are two cases:

- $\beta \notin Bel(X)$: then we have $\alpha \in Bel(X)$ and $\beta \notin Bel(X)$, showing that the conditions of the lemma are not satisfied.
- $\beta \in Bel(X)$: then it follows from $X \notin K^{\uparrow\perp\!\!\perp}\beta$ that there is some X' such that $X' \subset X$ and $\beta \in Bel(X')$. It follows from $X' \subset X \in K^{\uparrow\perp\!\!\perp}\alpha$ that $\alpha \notin Bel(X')$. We than have $\beta \in Bel(X')$ and $\alpha \notin Bel(X')$, showing that the conditions of the lemma are not satisfied.

Theorem 1. Let $K_{A_i} \in \mathcal{K}$ and let '$-_{\sigma_\downarrow}$' be a contraction operator. '$-_{\sigma_\downarrow}$' is an *optimal contraction using plausibility* for K_{A_i} if and only if it satisfies CP-1,..., CP-4, *that is*, it satisfies *success*, *inclusion*, *uniformity* and *minimal plausibility change*.

Proof
• *Postulates to Construction*. We need to show that if an operator ($-$) satisfies the enumerated postulates, then it is possible to build an operator in the way specified in the theorem ($-_{\sigma_\downarrow}$). Let '$\sigma_\downarrow$' be a function such that, for every base K_{A_i} ($K_{A_i} \in \mathcal{K}$) and for every consistent belief α, it holds that:

[Hypothesis] $\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha) = K^{\uparrow}_{A_i} \backslash K_{A_i} - \alpha.$

We must show:
– Part A.

1. '$\sigma_\downarrow$' is a well defined function.
2. $\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha) \subseteq \bigcup(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)$.
3. If $X \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha$, $X \neq \emptyset$, then $X \cap \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha) \neq \emptyset$.
4. If $(\beta, A_j) \in \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha)$ then $(\beta, A_j) \in X \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha$ and for all $(\delta, A_k) \in X$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

– Part B. '$-_{\sigma_\downarrow}$' is equal to '$-$', that is, $K_{A_i} -_{\sigma_\downarrow} \alpha = K_{A_i} - \alpha$.

Part A.
1. '$\sigma_\downarrow$' is a well defined function.

Let α and β two sentences such that $K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha = K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\beta$. We need to show that $\sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha) = \sigma_\downarrow(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\beta)$. It follows from $K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\alpha = K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\beta$, by Lemma 1, for all subset K' of $K^{\uparrow}_{A_i}$, $\alpha \in Bel(K')$ if and only if $\beta \in Bel(K')$. Since $Sen(K_{A_i}) = Sen(K^{\uparrow}_{A_i})$ and $K^{\uparrow}_{A_i} \subseteq K_{A_i}$ then, for all subset K'' of K_{A_i}, $\alpha \in Bel(K'')$ if and only if $\beta \in Bel(K'')$. Thus, by **uniformity**, $K_{A_i} - \alpha = K_{A_i} - \beta$. Therefore, by the definition of $\sigma_\downarrow$ adopted in the hypothesis, $\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha) = \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \beta)$.

2. $\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha) \subseteq \bigcup(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$.

Let $(\beta, A_j) \in \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$. By the definition of $\sigma_\downarrow$ adopted in the hypothesis $(\beta, A_j) \in (K^\uparrow_{A_i} - K_{A_i} - \alpha)$. Thus, $(\beta, A_j) \in K^\uparrow_{A_i}$ and $(\beta, A_j) \notin K_{A_i} - \alpha$. It follows by **minimal plausibility change** that there is some $K' \subseteq K^\uparrow_{A_i}$ such that $\alpha \notin Bel(K')$ and $\alpha \in Bel(K' \cup \{(\beta, A_j)\})$. By *compactness*, there is some finite subset K'' of K' such that $\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$. Since $\alpha \notin Bel(K')$ we have $\alpha \notin Bel(K'')$. It follows from $\alpha \notin Bel(K'')$ and $\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$ that there is some α-kernel that contains (β, A_j). Hence, $(\beta, A_j) \in \bigcup(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$.

3. If $X \in K^\uparrow_{A_i} \perp\!\!\!\perp \alpha$, $X \neq \emptyset$, then $X \cap \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha) \neq \emptyset$.

Let $\emptyset \neq X \in K^\uparrow_{A_i} \perp\!\!\!\perp \alpha$, we need to show that $X \cap \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha) \neq \emptyset$. We should prove that, there exists $(\beta, A_j) \in X$ such that $(\beta, A_j) \in \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$. By **success**, $\alpha \notin Bel(K_{A_i} - \alpha)$. Since $X \neq \emptyset$ then $\alpha \in Bel(X)$ and $X \nsubseteq K_{A_i} - \alpha$; that is, there is some (β, A_j) such that $(\beta, A_j) \in X$ and $(\beta, A_j) \notin K_{A_i} - \alpha$. Since $X \subseteq K^\uparrow_{A_i}$ it follows that $(\beta, A_j) \in (K^\uparrow_{A_i} - K_{A_i} - \alpha)$; that is, by the definition of $\sigma_\downarrow$ adopted in the hypothesis $(\beta, A_j) \in \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$. Therefore, $X \cap \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha) \neq \emptyset$.

4. If $(\beta, A_j) \in \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$ then $(\beta, A_j) \in X \in K^\uparrow_{A_i} \perp\!\!\!\perp \alpha$ and for all $(\delta, A_k) \in X$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

Suppose that $(\beta, A_j) \in \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$. Then, by the definition of $\sigma_\downarrow$ adopted in the hypothesis, $(\beta, A_j) \in (K^\uparrow_{A_i} \backslash K_{A_i} - \alpha)$. Thus, $(\beta, A_j) \in K^\uparrow_{A_i}$ and $(\beta, A_j) \notin K_{A_i} - \alpha$. It follows by **minimal plausibility change** that there is some $K' \subseteq K^\uparrow_{A_i}$ such that $\alpha \notin Bel(K')$, but $\alpha \in Bel(K' \cup \{(\beta, A_j)\})$ and for all $(\delta, A_k) \in K'$ such that $\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) - \{(\delta, A_k)\})$ it holds that $A_j \leq^{A_i}_{Co} A_k$. By *compactness*, there is some finite subset K'' of K' such that $\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$. Since $\alpha \notin Bel(K')$ we have $\alpha \notin Bel(K'')$. It follows from $\alpha \notin Bel(K'')$ and $\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$ that there is some α-kernel X that contains (β, A_j). Then, for all $(\delta, A_k) \in X$, $\alpha \notin Bel(X - \{(\delta, A_k)\})$. Since $X \subseteq K'$, it follows that $A_j \leq^{A_i}_{Co} A_k$.

Part B. '$-_{\sigma_\downarrow}$' is equal to '$-$', that is, $K_{A_i} -_{\sigma_\downarrow} \alpha = K_{A_i} - \alpha$.

Let '$-_{\sigma_\downarrow}$' a contraction operator defined as $K_{A_i} -_{\sigma_\downarrow} \alpha = K_{A_i} \backslash X$ where: $X = \{(\omega, A_j) : \omega \in Sen(\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha))$ and $(\omega, A_j) \in K_{A_i}\}$ and $\sigma_\downarrow$ defined as in the hypothesis.

($\supseteq$) Let $(\delta, A_j) \in K_{A_i} - \alpha$. It follows by **inclusion** that $K_{A_i} - \alpha \subseteq K_{A_i}$ and $(\delta, A_j) \in K_{A_i}$. Thus, it follows from $(\delta, A_j) \in K_{A_i} - \alpha$ and $(\delta, A_j) \in K_{A_i}$ that $(\delta, A_j) \notin (K_{A_i} \backslash K_{A_i} - \alpha)$. Since $K^\uparrow_{A_i} \subseteq K_{A_i}$, then $(\delta, A_j) \notin (K^\uparrow_{A_i} \backslash K_{A_i} - \alpha)$. Thus, by the definition of $\sigma_\downarrow$ adopted in the hypothesis, $(\delta, A_j) \notin \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$. We have two cases:

- $(\delta, A_j) \in K^\uparrow_{A_i}$. Then $\delta \notin Sen(\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha))$.
- $(\delta, A_j) \in K_{A_i}$. Then, if $(\delta, A_k) \in K^\uparrow_{A_i}$ it holds that $A_j \leq^{A_i}_{Co} A_k$. By *reductio ad absurdum*, suppose that $(\delta, A_k) \in \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$. Then $(\delta, A_k) \in (K^\uparrow_{A_i} \backslash K_{A_i} - \alpha)$ by the definition of $\sigma_\downarrow$ adopted in the hypothesis. Then $(\delta, A_k) \notin K_{A_i} - \alpha$ which is absurd due to we supposed that $(\delta, A_j) \in K_{A_i} - \alpha$ and $(\delta, A_k) \in K^\uparrow_{A_i}$. Therefore, $(\delta, A_k) \notin \sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha)$ and $\delta \notin Sen(\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha))$.

Hence, it follows from definition that $(\delta, A_j) \in K_{A_i} -_{\sigma_\downarrow} \alpha$.

($\subseteq$) Let $(\delta, A_j) \in K_{A_i} -_{\sigma_\downarrow} \alpha$. By definition $(\delta, A_j) \in K_{A_i} - X$ where $X = \{(\omega, A_k) : \omega \in Sen(\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha))$ and $(\omega, A_k) \in K_{A_i}\}$. Then, $(\delta, A_j) \in K_{A_i}$ and $(\delta, A_j) \notin X$. Therefore, $\delta \notin Sen(\sigma_\downarrow(K^\uparrow_{A_i} \perp\!\!\!\perp \alpha))$. Thus, by the definition of $\sigma_\downarrow$ adopted in the hypothesis, $\delta \notin Sen(K^\uparrow_{A_i} \backslash (K_{A_i} - \alpha))$. Hence, $\delta \in Sen(K_{A_i} - \alpha)$ and it must be the case in which $(\delta, A_j) \in K_{A_i} - \alpha$.

• *Construction to Postulates*. Let $-_{\sigma_\downarrow}$ be an optimal contraction using plausibility for K_{A_i}. We need to show that it satisfies the four conditions of the theorem.

(CP-1) *Success:* if $\alpha \notin Cn(\emptyset)$, then $\alpha \notin Bel(K_{A_i} -_{\sigma_\downarrow} \alpha)$.

Proof. Suppose to the contrary that $\alpha \notin Cn(\emptyset)$ and $\alpha \in Bel(K_{A_i} -_{\sigma_\downarrow} \alpha)$. By compactness, there is a finite subset K' of $K_{A_i} -_{\sigma_\downarrow} \alpha$ such that $\alpha \in Bel(K')$. There is then an α-kernel K'' such that $K'' \subseteq K'$. Since $K' \subseteq K_{A_i} -_{\sigma_\downarrow} \alpha \subseteq K_{A_i}$, K'' is also an α-kernel of K_{A_i}. We then have $K'' \in K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha$ and $K'' \subseteq K_{A_i} -_{\sigma_\downarrow} \alpha$. However, it follows from $\alpha \notin Cn(\emptyset)$ that $K'' \neq \emptyset$. By clause (2) of Definition 17, there is some $\beta \in Sen(K'')$ such that $\beta \in Sen(\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha))$. By Definition 20, $\beta \notin Sen(K_{A_i} -_{\sigma_\downarrow} \alpha)$, contrary to $\beta \in Sen(K'')$ with $K'' \subseteq K_{A_i} -_{\sigma_\downarrow} \alpha$.

(CP-2) *Inclusion:* $K_{A_i} -_{\sigma_\downarrow} \alpha \subseteq K_{A_i}$.
Proof. Straightforward by definition.

(CP-3) *Uniformity:* If for all $K' \subseteq K_{A_i}$, $\alpha \in Bel(K')$ if and only if $\beta \in Bel(K')$ then $K_{A_i} -_{\sigma_\downarrow} \alpha = K_{A_i} -_{\sigma_\downarrow} \beta$.

Proof. Suppose that for all subset K' of K_{A_i}, $\alpha \in Bel(K')$ if and only if $\beta \in Bel(K')$. By Lemma 1, $K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha = K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \beta$. Since '$\sigma_\downarrow$' is a well defined function then $\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha) = \sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \beta)$. Therefore, by Definition 20 $K_{A_i} -_{\sigma_\downarrow} \alpha = K_{A_i} -_{\sigma_\downarrow} \beta$.

(CP-4) *Minimal Plausibility Change*: If $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} -_{\sigma_\downarrow} \alpha$ then there is $K' \subseteq K_{A_i}$ where $\alpha \notin Bel(K')$ but there exists $(\beta, A_j) \in K_{A_i}$ such that:

- $\alpha \in Bel(K' \cup \{(\beta, A_j)\})$,
- $p = j$ or $A_p \leq^{A_i}_{Co} A_j$, and
- for all $(\delta, A_k) \in K'$ such that $\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) - \{(\delta, A_k)\})$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

Proof. Suppose $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} -_{\sigma_\downarrow} \alpha$. Then, by Definition 20, $(\beta, A_p) \in Y$ where $Y = \{(\omega, A_q) : \omega \in Sen(\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha)) \textit{ and } (\omega, A_q) \in K_{A_i}\}$. Then $\beta \in Sen(\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha))$. By Definition 18 of bottom incision function, $\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha) \subseteq \bigcup(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha)$, so that there is some information object (β, A_j) such that $(\beta, A_j) \in X \in K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \alpha$. It follows from Definition 7 that $(\beta, A_j) \in K^{\uparrow}_{A_i}$. Thus, $p = j$ or $A_p \leq^{A_i}_{Co} A_j$. Let $K' \subseteq K_{A_i}$ such that $X - \{(\beta, A_j)\} \subseteq K'$. We have two cases:

- $K' = X \setminus \{(\beta, A_j)\}$. Then, since X is minimal, $\alpha \notin Bel(K')$ but $\alpha \in Bel(K' \cup \{(\beta, A_j)\})$, and for all $(\delta, A_k) \in K'$, $\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) - \{(\delta, A_k)\})$. Therefore, by Definition 18, $A_j \leq^{A_i}_{Co} A_k$.
- $X \setminus \{(\beta, A_j)\} \subset K'$ and $\{(\beta, A_j)\} \notin K'$. Then there exists $(\delta, A_k) \in K'$ such that $A_k <^{A_i}_{Co} A_j$ and, by Definition 18, $(\delta, A_k) \notin X$. Therefore, $\alpha \in Bel((K' \cup \{(\beta, A_j)\}) - \{(\delta, A_k)\})$.

Theorem 2. Let $K_{A_i} \in \mathcal{K}$ and let '$*_{\sigma_\downarrow}$' be a revision operator. '$*_{\sigma_\downarrow}$' is a *prioritized revision using plausibility* for K_{A_i} if and only if it satisfies RP-1,…, RP-5, *that is*, it satisfies *success*, *inclusion*, *consistency*, *uniformity* and *minimal plausibility change*.

Proof
• *Postulates to Construction*. We need to show that if an operator ($*$) satisfies the enumerated postulates, then it is possible to build an operator in the way specified in the theorem ($*_{\sigma_\downarrow}$). Let '$\sigma_\downarrow$' be a function such that, for every base K_{A_i} ($K_{A_i} \in \mathcal{K}$) and for every consistent belief α, it holds that:

[Hypothesis] $\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha) = K^{\uparrow}_{A_i} - K_{A_i} * (\alpha, A_j)$.

We must show:
– Part A.

1. '$\sigma_\downarrow$' is a well defined function.
2. $\sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha) \subseteq \bigcup(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha)$.
3. If $X \in K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha$, $X \neq \emptyset$, then $X \cap \sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha) \neq \emptyset$.
4. If $(\beta, A_j) \in \sigma_\downarrow(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha)$ then $(\beta, A_j) \in X \in K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha$ and for all $(\delta, A_k) \in X$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

– Part B. '$*_{\sigma_\downarrow}$' is equal to '$*$', that is, $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) = K_{A_i} * (\alpha, A_j)$.

Part A.

1. '$\sigma_{\downarrow}$' is a well defined function.

Let $\neg\alpha$ and $\neg\beta$ two sentences such that $K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha = K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\beta$. We need to show that $\sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha) = \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\beta)$. It follows from $K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha = K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\beta$, by Lemma 1, for all subset K' of $K^{\uparrow}_{A_i}$, $\neg\alpha \in Bel(K')$ if and only if $\neg\beta \in Bel(K')$. That is, for all subset K' of $K^{\uparrow}_{A_i}$, $Sen(K') \vdash \neg\alpha$ if and only if $Sen(K') \vdash \neg\beta$. Then, for all subset K' of $K^{\uparrow}_{A_i}$, $\{\alpha\}(K') \vdash \perp$ if and only if $\{\beta\}(K') \vdash \perp$. Since, $K^{\uparrow}_{A_i} \subseteq K_{A_i}$ (I) and $Sen(K^{\uparrow}_{A_i}) = Sen(K_{A_i})$, then for all subset K'' of K_{A_i}, $\{\alpha\}(K'') \vdash \perp$ if and only if $\{\beta\}(K'') \vdash \perp$. Thus, by **uniformity**, $K_{A_i} \cap (K_{A_i} * (\alpha, A_j)) = K_{A_i} \cap (K_{A_i} * (\beta, A_k))$. Then, $K_{A_i} \backslash (K_{A_i} * (\alpha, A_j)) = K_{A_i} - (K_{A_i} * (\beta, A_k))$ (II). Therefore, by definition of $\sigma_{\downarrow}$ adopted in the hypothesis, (I) and (II), then $\sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha) = \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\beta)$.

2. $\sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha) \subseteq \bigcup(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$.

Let $(\beta, A_j) \in \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$. By the definition of $\sigma_{\downarrow}$ adopted in the hypothesis $(\beta, A_j) \in (K^{\uparrow}_{A_i} \backslash K_{A_i} * (\alpha, A_k))$. Thus, $(\beta, A_j) \in K^{\uparrow}_{A_i}$ and $(\beta, A_j) \notin K_{A_i} * (\alpha, A_k)$. It follows by **minimal plausibility change** that there is some $K' \subseteq K^{\uparrow}_{A_i}$ such that $\neg\alpha \notin Bel(K')$ and $\neg\alpha \in Bel(K' \cup \{(\beta, A_j)\})$. By *compactness*, there is some finite subset K'' of K' such that $\neg\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$. Since $\neg\alpha \notin Bel(K')$ we have $\neg\alpha \notin Bel(K'')$. It follows from $\neg\alpha \notin Bel(K'')$ and $\neg\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$ that there is some $\neg\alpha$-kernel that contains (β, A_j). Hence $(\beta, A_j) \in \bigcup(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$.

3. If $X \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha$, $X \neq \emptyset$, then $X \cap \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha) \neq \emptyset$.

Let $\emptyset \neq X \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha$, we need to show that $X \cap \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha) \neq \emptyset$. We should prove that, there exists $(\beta, A_j) \in X$ such that $(\beta, A_j) \in \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$. Suppose that α is consistent. Since we have assumed that K_{A_i} is consistent, by **consistency**, $K_{A_i} * (\alpha, A_k)$ is consistent. Since $X \neq \emptyset$ and X is inconsistent with α then $X \nsubseteq K_{A_i} * (\alpha, A_k)$ by **success**. This means that there is some $(\beta, A_j) \in X$ and $(\beta, A_j) \notin K_{A_i} * (\alpha, A_k)$. Since $X \subseteq K^{\uparrow}_{A_i}$ it follows that $(\beta, A_j) \in (K^{\uparrow}_{A_i} \backslash K_{A_i} * (\alpha, A_k))$; that is, by the definition of $\sigma_{\downarrow}$ adopted in the hypothesis $(\beta, A_j) \in \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$. Therefore, $X \cap \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha) \neq \emptyset$.

4. If $(\beta, A_j) \in \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$ then $(\beta, A_j) \in X \in K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha$ and for all $(\delta, A_k) \in X$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

Suppose that $(\beta, A_j) \in \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$. Then, by the definition of $\sigma_{\downarrow}$ adopted in the hypothesis $(\beta, A_j) \in (K^{\uparrow}_{A_i} - K_{A_i} * (\alpha, A_p))$. Thus, $(\beta, A_j) \in K^{\uparrow}_{A_i}$ and $(\beta, A_j) \notin K_{A_i} * (\alpha, A_p)$. It follows by **minimal plausibility change** that there is some $K' \subseteq K^{\uparrow}_{A_i}$ such that $\neg\alpha \notin Bel(K')$, but $\neg\alpha \in Bel(K' \cup \{(\beta, A_j)\})$ and for all $(\delta, A_k) \in K'$ such that $\neg\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) - \{(\delta, A_k)\})$ it holds that $A_j \leq^{A_i}_{Co} A_k$. By *compactness*, there is some finite subset K'' of K' such that $\neg\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$. Since $\neg\alpha \notin Bel(K')$ we have $\neg\alpha \notin Bel(K'')$. It follows from $\neg\alpha \notin Bel(K'')$ and $\neg\alpha \in Bel(K'' \cup \{(\beta, A_j)\})$ that there is some $\neg\alpha$-kernel X that contains (β, A_j). Then, for all $(\delta, A_k) \in X$, $\neg\alpha \notin Bel(X \backslash \{(\delta, A_k)\})$. Since $X \subseteq K'$, it follows that $A_j \leq^{A_i}_{Co} A_k$.

Part B. '$*_{\sigma_{\downarrow}}$' is equal to '$*$', that is, $K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j) = K_{A_i} * (\alpha, A_j)$.

Let '$*_{\sigma_{\downarrow}}$' a revision operator defined as $K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j) = (K_{A_i} - X) \cup \{(\alpha, A_j)\}$ where: $X = \{(\omega, A_p) : \omega \in Sen(\sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha))\text{and}(\omega, \text{A}_\text{p}) \in \text{K}_{\text{A}_\text{i}}\}$ and $\sigma_{\downarrow}$ defined as in the hypothesis.

($\supseteq$) Let $(\delta, A_k) \in K_{A_i} * (\alpha, A_j)$. It follows by **inclusion** that $K_{A_i} * (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$ and $(\delta, A_k) \in K_{A_i} \cup \{(\alpha, A_j)\}$. If $(\delta, A_k) = (\alpha, A_j)$ then $(\delta, A_k) \in K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)$ by definition. Suppose that $(\delta, A_k) \neq (\alpha, A_j)$. Since $(\delta, A_k) \in K_{A_i} \cup \{(\alpha, A_j)\}$ then $(\delta, A_k) \in K_{A_i}$. Thus, it follows from $(\delta, A_k) \in K_{A_i} * (\alpha, A_j)$ and $(\delta, A_k) \in K_{A_i}$ that $(\delta, A_k) \notin (K_{A_i} \backslash K_{A_i} * (\alpha, A_j))$. Since $K^{\uparrow}_{A_i} \subseteq K_{A_i}$ then $(\delta, A_k) \notin (K^{\uparrow}_{A_i} - K_{A_i} * (\alpha, A_j))$. Therefore, by the definition of $\sigma_{\downarrow}$ adopted in the hypothesis, $(\delta, A_k) \notin \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$. We have two cases:

- $(\delta, A_k) \in K^{\uparrow}_{A_i}$. Then $\delta \notin Sen(\sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha))$.
- $(\delta, A_k) \in K_{A_i}$. Then, if $(\delta, A_p) \in K^{\uparrow}_{A_i}$ it holds that $A_k \leq^{A_i}_{Co} A_p$. By *reductio ad absurdum*, suppose that $(\delta, A_p) \in \sigma_{\downarrow}(K^{\uparrow}_{A_i}{}^{\perp\!\!\perp}\neg\alpha)$ then $(\delta, A_p) \in (K^{\uparrow}_{A_i} \backslash K_{A_i} * (\alpha, A_j))$ by the definition of $\sigma_{\downarrow}$ adopted in

the hypothesis. Then $(\delta, A_p) \notin K_{A_i} * (\alpha, A_j)$ which is absurd due to we supposed that $(\delta, A_k) \in K_{A_i} * (\alpha, A_j)$ and $(\delta, A_p) \in K^{\uparrow}_{A_i}$. Therefore $(\delta, A_p) \notin \sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha)$. Thus $\delta \notin Sen(\sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha))$.

Therefore, it follows from definition that $(\delta, A_k) \in K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)$.

($\subseteq$) Let $(\delta, A_k) \in K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)$. It follows from definition that $K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$ and $(\delta, A_k) \in K_{A_i} \cup \{(\alpha, A_j)\}$. Then $(\delta, A_k) \in K_{A_i}$ or $(\delta, A_k) = (\alpha, A_j)$. We have two cases:

- $(\delta, A_k) = (\alpha, A_j)$. Then, by **success**, $(\delta, A_k) \in K_{A_i} * (\alpha, A_j)$.
- $(\delta, A_k) \in K_{A_i}$ and $(\delta, A_k) \neq (\alpha, A_j)$. Then, by definition, $\delta \notin Sen(\sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha))$. By the definition of $\sigma_{\downarrow}$ adopted in the hypothesis, then $\delta \notin Sen(K^{\uparrow}_{A_i} \setminus (K_{A_i} * (\alpha, A_j))$. From $(\delta, A_k) \in K_{A_i}$ then $\delta \in Sen(K_{A_i})$. Since $Sen(K_{A_i}) = Sen(K^{\uparrow}_{A_i})$ then $\delta \in Sen(K_{A_i} * (\alpha, A_j))$. Therefore, it must be the case in which $(\delta, A_k) \in K_{A_i} * (\alpha, A_j)$.

• *Construction to Postulates*. Let $*_{\sigma_{\downarrow}}$ be an prioritized revision using plausibility for K_{A_i}. We need to show that it satisfies the five conditions of the theorem.

(RP-1) *Success:* $(\alpha, A_j) \in K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)$.
Proof. Straightforward by definition.

(RP-2) *Inclusion:* $K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$.
Proof. Straightforward by definition.

(RP-3) *Consistency*: if α is consistent then $K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)$ is consistent.
Proof. Straightforward by definition.

(RP-4) *Uniformity:* If for all $K' \subseteq K_{A_i}$, $\{\alpha\} \cup Sen(K') \vdash \perp$ if and only if $\{\beta\} \cup Sen(K') \vdash \perp$ then $K_{A_i} \cap (K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)) = K_{A_i} \cap (K_{A_i} *_{\sigma_{\downarrow}} (\beta, A_k))$.
Proof. Suppose that for all subset K' of K_{A_i}, $\{\alpha\} \cup Sen(K') \vdash \perp$ if and only if $\{\beta\} \cup Sen(K') \vdash \perp$. Then, it follows that $Sen(K') \vdash \neg\alpha$ if and only if $Sen(K') \vdash \neg\beta$; that is, $\neg\alpha \in Bel(K')$ if and only if $\neg\beta \in Bel(K')$. Hence, by Lemma 1, $K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha = K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\beta$. Since '$\sigma_{\downarrow}$' is a well defined function then $\sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha) = \sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\beta)$. Therefore, by Definition 20, $K_{A_i} -_{\sigma_{\downarrow}} \neg\alpha = K_{A_i} -_{\sigma_{\downarrow}} \neg\beta$. Then, by Definition 21, $K_{A_i} \cap (K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_j)) = K_{A_i} \cap (K_{A_i} *_{\sigma_{\downarrow}} (\beta, A_k))$.

(RP-5) *Minimal Plausibility Change*: If $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_k)$ then there is $K' \subseteq K_{A_i}$ where $\neg\alpha \notin Bel(K')$ but there exists $(\beta, A_j) \in K_{A_i}$ such that:

- $\neg\alpha \in Bel(K' \cup \{(\beta, A_j)\})$,
- $p = j$ or $A_p \leq^{A_i}_{Co} A_j$, and
- for all $(\delta, A_k) \in K'$ such that $\neg\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) \setminus \{(\delta, A_k)\})$ it holds that $A_j \leq^{A_i}_{Co} A_k$.

Proof. Suppose $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} *_{\sigma_{\downarrow}} (\alpha, A_k)$. Then, by Definition 21, $(\beta, A_p) \notin (K_{A_i} -_{\sigma_{\downarrow}} \neg\alpha) + (\alpha, A_k)$. Thus, by Definition 20, $(\beta, A_p) \in \{(\omega, A_q) : \omega \in Sen(\sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha))$ *and* $(\omega, A_q) \in K_{A_i}\}$. Then $\beta \in Sen(\sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha))$. By Definition 18 of bottom incision function, $\sigma_{\downarrow}(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha) \subseteq \bigcup(K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha)$, so that there is some information object (β, A_j) such that $(\beta, A_j) \in X \in K^{\uparrow}_{A_i} {}^{\perp\!\!\perp} \neg\alpha$. It follows from Definition 7 that $(\beta, A_j) \in K^{\uparrow}_{A_i}$. Thus, $p = j$ or $A_p \leq^{A_i}_{Co} A_j$. Let $K' \subseteq K_{A_i}$ such that $X \setminus \{(\beta, A_j)\} \subseteq K'$. We have two cases:

- $K' = X \setminus \{(\beta, A_j)\}$. Then, since X is minimal, $\neg\alpha \notin Bel(K')$ but $\neg\alpha \in Bel(K' \cup \{(\beta, A_j)\})$, and for all $(\delta, A_k) \in K'$, $\neg\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) \setminus \{(\delta, A_k)\})$. Hence, by Definition 18, $A_j \leq^{A_i}_{Co} A_k$.
- $X \setminus \{(\beta, A_j)\} \subset K'$ and $\{(\beta, A_j)\} \notin K'$. Then, there exists $(\delta, A_k) \in K'$ such that $A_k <^{A_i}_{Co} A_j$ and, by Definition 18, $(\delta, A_k) \notin X$. Therefore, $\neg\alpha \in Bel((K' \cup \{(\beta, A_j)\}) \setminus \{(\delta, A_k)\})$.

Proposition 6. If '+' satisfies EP-1,…,EP-5 and '$-_{\sigma_{\downarrow}}$' satisfies CP-1,…,CP-4 then '$*_{\sigma_{\downarrow}}$' satisfies RP-1,…,RP-5.

Proof

Let '$*_{\sigma_\downarrow}$' be a prioritized revision using plausibility for K_{A_i}, defined as $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) = (K_{A_i} -_{\sigma_\downarrow} \neg\alpha) + (\alpha, A_j)$. We need to show that it satisfies RP-1,...,RP-5 from the postulates of expansion using plausibility and from the postulates of optimal contraction using plausibility.

(RP-1) *Success:* $(\alpha, A_j) \in K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j)$.
Proof. Straightforward by Definition 21 and EP-1.

(RP-2) *Inclusion:* $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$.
Proof. It follows from CP-2 that $K_{A_i} -_{\sigma_\downarrow} \neg\alpha \subseteq K_{A_i}$. Then, $(K_{A_i} -_{\sigma_\downarrow} \neg\alpha) \cup \{(\alpha, A_j)\} \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$. Thus, by Definition 16 $(K_{A_i} -_{\sigma_\downarrow} \neg\alpha) + (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$. Hence, by Definition 21, $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) \subseteq K_{A_i} \cup \{(\alpha, A_j)\}$.

(RP-3) *Consistency*: if α is consistent then $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j)$ is consistent.
Proof. By Definition 21, $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j) = (K_{A_i} -_{\sigma_\downarrow} \neg\alpha) + (\alpha, A_j)$. From EP-1 and CP-1 it follows that $K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j)$ is consistent.

(RP-4) *Uniformity:* If for all $K' \subseteq K_{A_i}$, $\{\alpha\} \cup Sen(K') \vdash \perp$ if and only if $\{\beta\} \cup Sen(K') \vdash \perp$ then $K_{A_i} \cap (K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j)) = K_{A_i} \cap (K_{A_i} *_{\sigma_\downarrow} (\beta, A_k))$.
Proof. Suppose that for all subset K' of K_{A_i}, $\{\alpha\} \cup Sen(K') \vdash \perp$ if and only if $\{\beta\} \cup Sen(K') \vdash \perp$. Then, it follows that $Sen(K') \vdash \neg\alpha$ if and only if $Sen(K') \vdash \neg\beta$; that is, $\neg\alpha \in Bel(K')$ if and only if $\neg\beta \in Bel(K')$. Thus, it follows from CP-3 that $K_{A_i} -_{\sigma_\downarrow} \neg\alpha = K_{A_i} -_{\sigma_\downarrow} \neg\beta$. Hence, by Definition 21, $K_{A_i} \cap (K_{A_i} *_{\sigma_\downarrow} (\alpha, A_j)) = K_{A_i} \cap (K_{A_i} *_{\sigma_\downarrow} (\beta, A_k))$.

(RP-5) *Minimal Plausibility Change*: If $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} *_{\sigma_\downarrow} (\alpha, A_k)$ then there is $K' \subseteq K_{A_i}$ where $\neg\alpha \notin Bel(K')$ but there exists $(\beta, A_j) \in K_{A_i}$ such that:

- $\neg\alpha \in Bel(K' \cup \{(\beta, A_j)\})$,
- $p = j$ or $A_p \leq_{Co}^{A_i} A_j$, and
- for all $(\delta, A_k) \in K'$ such that $\neg\alpha \notin Bel((K' \cup \{(\beta, A_j)\}) - \{(\delta, A_k)\})$ it holds that $A_j \leq_{Co}^{A_i} A_k$.

Proof. Suppose $(\beta, A_p) \in K_{A_i}$ and $(\beta, A_p) \notin K_{A_i} *_{\sigma_\downarrow} (\alpha, A_k)$. Then, by Definition 21, $(\beta, A_p) \notin K_{A_i} -_{\sigma_\downarrow} \neg\alpha$. Thus, it follows by CP-4 that all conditions of the postulate are satisfied.

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