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RESEARCH ARTICLE

Wireless power transfer between one transmitter and two receivers: optimal analytical solution

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This paper focuses on non-radiative wireless power transfer implemented by means of a resonant magnetic coupling. The case of one transmitter and two receivers is considered and a rigorous analytical procedure is developed demonstrating that maximum power transfer or maximum efficiency can be achieved by appropriately selecting the load values. Both cases of coupled and uncoupled receivers are solved; closed formulas are derived for the optimal loads, which maximize either power or efficiency. It is shown that the resistances that realize maximum power transfer are always greater than the resistances that realize maximum efficiency. According to this observation, an optimal range of operation for the load resistances is also determined. Furthermore, it is demonstrated that in the case where the receivers are coupled the introduction of appropriate compensating reactances allows retrieving the same results corresponding to the uncoupled case both for powers and efficiency. Theoretical data are validated by comparisons with numerical results.

Keywords: Wireless power transmission, Magnetic coupling, Two receivers, Power maximization, Efficiency maximization

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I. INTRODUCTION

In the last years, reactive wireless power transfer (WPT) has received a growing interest; the research is mainly focused on wireless resonant energy links (WRELs) based on a magnetic coupling between resonators. Generally, WRELs are implemented by means of a single receiver direct feed, that is a wireless link achieved by a magnetic coupling between a transmitting resonator connected to the power generator and a single receiving resonator connected to the device to be powered [1–21]. With reference to these single receiver links, the problem of how to realize load and matching elements in such a way to maximize efficiency has been solved in [22, 23]. More recently, by using a two-port network representation, in [24] it has been demonstrated that the load impedances, which maximize efficiency are quite different from those required to maximize the power delivered to the load. Moreover, the approach has also provided the necessary reactive elements to obtain optimal performance.

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However, although the problem of a single receiver direct feed has been studied widely in the literature by deriving accurate analytical solutions, to date, very few contributions have dealt with the multiple receiver case [9, 25]. For instance, in [9] some experimental results demonstrating the feasibility of using a magnetic coupling to transmit power from a single transmitter to two small receivers is demonstrated. A theoretical analysis is reported in [25], where the problem of determining the load impedances for efficiency maximization has been solved in the case of multiple receivers. However, the analysis developed in [25] is limited to the case of uncoupled receivers and to efficiency maximization: no results are established for power maximization. According to these observations, the multiple receivers WREL system is a topic that still deserves further theoretical investigations.

To this regard, the following questions arise: theoretical results obtained for a single receiver WREL are still valid when a second receiver is added? In this case, which are the values of the loads that maximize either efficiency or the power delivered on the loads?

In this contribution, the answers to the above reported questions are provided. In particular, by analytical optimization, closed formulas for the optimal value of the loads for both power and efficiency maximization are derived for a two-receiver WREL system. It is demonstrated that these solutions are different from those obtained in the case of a single receiver.

The paper is organized as follows: in Section II we illustrate the problems we are referring to and we introduce the relevant notations. In Section III the theory is developed for two cases: the maximum power transfer case and the maximum efficiency case. In Section IV numerical examples illustrate the application of the developed theory.

II. PROBLEM DESCRIPTION AND TYPE OF DESIRED SOLUTION

A) Structure description

We consider a WREL system transmitting power by inductive coupling from a single transmitter to two receivers. As illustrated in Fig. 1, the structure can be schematized by means of three coupled inductors: L_i with $i = 1, 2, 3$. We assume that a generator is present at port 1, while at port 2 and 3 we have the loads R_2 and R_3, respectively. We also assume that a compensating capacitor C_i in series configuration with each inductor is present, so that the operating angular frequency ω_0 is: $\omega_0 = 1/\sqrt{C_i L_i}$. In addition, we take into account losses by means of an equivalent series resistances r_{ii} related to the respective quality factors Q_i by the standard relationships $Q_i = \omega_0 L_i / r_{ii}$.

In this paper we will consider the coupling between resonator i and the resonator j expressed by M_{ij} as

$$M_{ij} = k_{ij}\sqrt{L_i L_j}, \tag{1}$$

where the terms k_{ij} are the coupling coefficients and ω is the angular frequency. In the following part of this paper both the case of uncoupled receivers (i.e. $k_{23} = 0$) and the case where a coupling between the two receivers is present (i.e. $k_{23} \neq 0$) will be considered.

B) Desired solutions

Let us denote with P_2 and P_3 the active power delivered to the loads on port 2 and 3. We also denote with P_1 the active power supplied to the network by the voltage source V_1. We assume that the operating frequency is $\omega_0 = \sqrt{C_i L_i}$ and we look for two types of solutions:

- power maximization on the loads: find the values of R_2, R_3, which maximize the total power $P_{tot} = P_2 + P_3$;
- efficiency maximization: find the values of R_2, R_3, which maximize the efficiency η_{tot}:

$$\eta_{tot} = \eta_{12} + \eta_{13}, \tag{2}$$

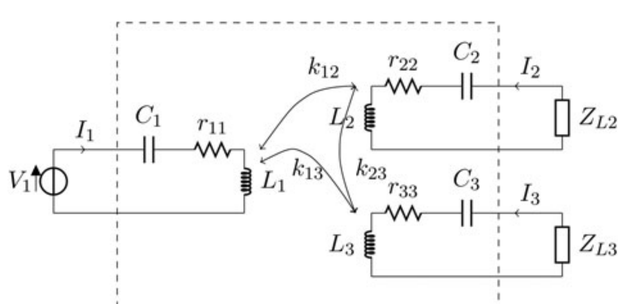

Fig. 1. Three-port network showing one transmitter at port 1 and two receivers at port 2 and 3.

where η_{12} and η_{13} are defined as follows:

$$\eta_{12} = \frac{P_2}{P_1}$$
$$\eta_{13} = \frac{P_3}{P_1}. \tag{3}$$

In order to find the desired solutions we will explicitly compute the relevant quantities and perform their derivatives with respect to R_2 and R_3. By setting these derivatives to zero we will find the sought value of load impedances.

III. POWER COMPUTATION AND OPTIMAL LOAD IMPEDANCES

The analysis of the circuit shown in Fig. 1 provides the following equations

$$V_1 = \left(r_{11} + j\omega L_1 + \frac{1}{j\omega C_1}\right)I_1 + j\omega M_{12} I_2 + j\omega M_{13} I_3$$

$$0 = j\omega M_{12} I_1 + \left(Z_{L2} + r_{22} + j\omega L_2 + \frac{1}{j\omega C_2}\right)I_2 + j\omega M_{23} I_3$$

$$0 = j\omega M_{13} I_1 + j\omega M_{23} I_2 + \left(Z_{L3} + r_{33} + j\omega L_3 + \frac{1}{j\omega C_3}\right)I_3$$

Accordingly, the three-port network inside the dashed lines in Fig. 1 can be represented by the relation

$$\mathbf{V} = \mathbf{Z_o I}, \tag{4}$$

where $\mathbf{V}$ is the vector of port voltages, $\mathbf{I}$ is the vector of port currents and $\mathbf{Z_o}$ is the impedance matrix, which is given by:

$$\mathbf{Z_o} = \begin{pmatrix} r_{11} + j\omega L_1\left(1 - \frac{\omega_0^2}{\omega^2}\right) & j\omega M_{12} & j\omega M_{13} \\ j\omega M_{12} & r_{22} + j\omega L_2\left(1 - \frac{\omega_0^2}{\omega^2}\right) & j\omega M_{23} \\ j\omega M_{13} & j\omega M_{23} & r_{33} + j\omega L_3\left(1 - \frac{\omega_0^2}{\omega^2}\right) \end{pmatrix}. \tag{5}$$

It is convenient to introduce the quantities x_{ij} defined as:

$$x_{ij} = \omega M_{ij}. \tag{6}$$

By using this definition, at the frequency of resonance ω_0, $\mathbf{Z_o}$ assumes the following expression:

$$\mathbf{Z_o} = \begin{pmatrix} r_{11} & jx_{12} & jx_{13} \\ jx_{12} & r_{22} & jx_{23} \\ jx_{13} & jx_{23} & r_{33} \end{pmatrix}. \tag{7}$$

The vector of port voltages can be expressed as follows:

$$\mathbf{V} = \begin{pmatrix} V_1 \\ -R_2 I_2 \\ -R_3 I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{pmatrix} \mathbf{I} \doteq \mathbf{E} - \mathbf{Z_L I}, \quad (8)$$

with obvious meaning of the symbols. By substitution into (4) we obtain:

$$\mathbf{E} - \mathbf{Z_L I} = \mathbf{Z_o I}, \quad (9)$$

or

$$\mathbf{I} = (\mathbf{Z_L} + \mathbf{Z_o})^{-1}\mathbf{E} = \mathbf{YE}, \quad (10)$$

where we have introduced the admittance matrix $\mathbf{Y}$.

We denote with P_1 the active power supplied by the voltage source; the active power absorbed by load i is denoted with P_i and is given by

$$P_i = \frac{1}{2} R_i I_i I_i^* = \frac{|V_1|^2}{2} R_i y_{i1} y_{i1}^*, \; i = 2, 3, \quad (11)$$

where we have denoted with y_{ij} the elements of the admittance matrix $\mathbf{Y}$. It is convenient to introduce the following quantity

$$\Delta = r_{11}R_2R_3 + r_{11}r_{22}R_3 + x_{12}^2 R_3 + r_{11}r_{33}R_2 + x_{13}^2 R_2 + r_{11}r_{22}r_{33} + x_{12}^2 r_{33} + x_{13}^2 r_{22}, \quad (12)$$

which allows to write the terms in the first column of the admittance matrix as

$$\begin{aligned} y_{11} &= \frac{(R_2 + r_{22})(R_3 + r_{33})}{\Delta} \\ y_{21} &= j\frac{x_{12}(R_3 + r_{33})}{\Delta} \\ y_{31} &= j\frac{x_{13}(R_2 + r_{22})}{\Delta}. \end{aligned} \quad (13)$$

It is also convenient to introduce the normalized powers P_i defined as

$$p_i = \frac{8 r_{11} P_i}{|V_1|^2}. \quad (14)$$

With the above definitions the power values are given by

$$\begin{aligned} p_1 &= 4r_{11}\frac{(R_2 + r_{22})(R_3 + r_{33})}{\Delta} \\ p_2 &= 4r_{11}\frac{x_{12}^2 R_2 (R_3 + r_{33})^2}{\Delta^2} \\ p_3 &= 4r_{11}\frac{x_{13}^2 (R_2 + r_{22})^2 R_3}{\Delta^2}. \end{aligned} \quad (15)$$

A) Maximum power solution

In this section the problem of power maximization is addressed. The value of the load impedances, which allows to maximize the power delivered at port 2 and 3 is determined both in the case of coupled and uncoupled receivers. Note that in the case of coupled receivers the analytical complexity prevents to solve the problem in closed form. Nonetheless, by using appropriate reactive elements we can still retrieve the same results as in the uncoupled case.

1) THE CASE OF UNCOUPLED RECEIVERS

We start our analysis by considering the case, where no coupling is present between the two receivers (i.e. resonators 2 and 3). In this case, the terms $Z_{o,23}$ and $Z_{o,32}$ of the impedance matrix $\mathbf{Z_o}$ are equal to zero (i.e. $x_{23} = 0$), and (7) simplifies as follows:

$$\mathbf{Z_o} = \begin{pmatrix} r_{11} & jx_{12} & jx_{13} \\ jx_{12} & r_{22} & 0 \\ jx_{13} & 0 & r_{33} \end{pmatrix}. \quad (16)$$

In order to better express the solutions it is convenient to introduce the following definitions:

$$\begin{aligned} \chi_{12}^2 &= \frac{x_{12}^2}{r_{11} r_{22}} = k_{12}^2 Q_1 Q_2 \\ \chi_{13}^2 &= \frac{x_{13}^2}{r_{11} r_{33}} = k_{13}^2 Q_1 Q_3 \\ \chi_{23}^2 &= \frac{x_{23}^2}{r_{22} r_{33}} = k_{23}^2 Q_2 Q_3 \\ \theta &= \sqrt{\left(1 + \chi_{12}^2 + \chi_{13}^2\right)}. \end{aligned} \quad (17)$$

It is feasible, by using a computer algebra system, to perform the derivatives of the total power $P_{tot} = P_2 + P_3$ with respect to R_2, R_3, equating them to zero and solving the system. The rather long expressions are not reported here. By doing this procedure we have obtained six solutions, but only one is meaningful since five solutions refer to negative resistance values. The load resistances that realize maximum power transfer are:

$$\begin{aligned} R_2' &= r_{22}\,\theta^2 \\ R_3' &= r_{33}\,\theta^2, \end{aligned} \quad (18)$$

where we have used the prime in order to refer to the maximum power solutions. It is noted that this result nicely extends the one obtained in [24]. Note that when the results in (18) are inserted into (13) we recover the following expressions

$$\begin{aligned} y_{11} &= \frac{\theta^2 + 1}{2 r_{11} \theta^2} \\ y_{21} &= -j\frac{\chi_{12}}{2\sqrt{r_{11} r_{22}}\theta^2} \\ y_{31} &= -j\frac{\chi_{13}}{2\sqrt{r_{11} r_{33}}\theta^2}. \end{aligned} \quad (19)$$

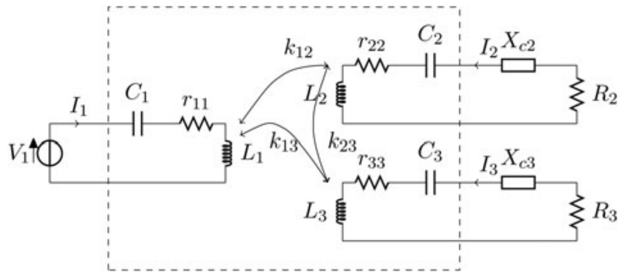

Fig. 2. When the cross-coupling term k_{23} is different from zero, it is still possible to add the compensating reactances X_{c2}, X_{c3} that allows to retrieve the same results of the uncoupled case. The compensating reactances are the same for the maximum efficiency and maximum power transfer cases so that $X''_{c2} = X'_{c2} = X_{c2}$, $X''_{c3} = X'_{c3} = X_{c3}$.

2) THE CASE OF COUPLED RECEIVERS

Let us now consider the case of coupled receivers (i.e. $k_{23} \neq 0$). This case differs from the previous one because of the terms $Z_{o,23}$ and $Z_{o,32}$ of the impedance matrix $\mathbf{Z_o}$ that are now different from zero and depend on the coupling between port 2 and port 3. In more detail, in the coupled case the three ports network is described by the impedance matrix $\mathbf{Z_o}$ as given in (7); it can be seen that at the frequency of resonance $Z_{o,23}$ and $Z_{o,32}$ are purely imaginary. As a consequence, we can infer that:

- the values of the resistances R'_2, R'_3 remain the same as found for the uncoupled case;
- we need to add two compensating series reactances X'_{c2}, X'_{c3}, as shown in Fig. 2, in order to maximize power.

In particular, the values of these compensating reactances are:

$$X'_{c2} = -\frac{x_{13}x_{23}}{x_{12}}\frac{r_{22}}{r_{33}}$$
$$X'_{c3} = -\frac{x_{12}x_{23}}{x_{13}}\frac{r_{33}}{r_{22}}. \quad (20)$$

Note that when inserting the above elements in our network we recover again the admittances reported in (19). In fact, from a practical point of view, the purpose of these reactances is to compensate for the presence of a coupling between the two receivers, thus making the coupled case identical to the uncoupled one; so that, we recover the same solution for the load impedances. By using the values given in (18) and in (20) we get the corresponding input impedance, normalized powers and efficiencies as reported in Table 1.

B) Maximum efficiency solution

In this section we look for the load impedances R_2 and R_3, which allow to maximize the efficiency as defined in (2).

1) THE CASE OF UNCOUPLED RECEIVERS

This time, using again a computer algebra system, we have computed the derivatives of the efficiency defined in (2) with respect to R_2, R_3; we have equated them to zero and solved the system. The rather long expressions are not reported here. By doing this procedure we have obtained three possible solutions, with only one meaningful, since the other two solutions refer to negative impedance values. The load impedances, which maximize efficiency are:

$$R''_2 = r_{22}\theta$$
$$R''_3 = r_{33}\theta, \quad (21)$$

where we have used the double prime in order to refer to the maximum efficiency solution. It is noted that also this result extends the one obtained in [24]. When using the values of (21) into (13) we obtain

$$y_{11} = \frac{1}{r_{11}\theta^2}$$
$$y_{21} = -j\frac{\chi_{12}}{2\sqrt{r_{11}r_{22}}(\theta(\theta+1))}$$
$$y_{31} = -j\frac{\chi_{13}}{2\sqrt{r_{11}r_{33}}(\theta(\theta+1))}. \quad (22)$$

2) THE CASE OF COUPLED RECEIVERS

We are not able to solve the maximum efficiency case when $x_{23} \neq 0$. In this regard, it is worth observing that in [24] it was demonstrated that the compensating reactances to be added for maximum power or for maximum efficiency, are the same. Can we extend this result to the two receivers case? When we insert the compensating reactances given in (20) we retrieve the same results for powers in both the coupled and the uncoupled case, is this true also for the efficiency?

It can be demonstrated that the answer is positive for the special cases of the loads corresponding to maximum efficiency and maximum power solution. More in general, it can be demonstrated that the compensating reactances allow to retrieve the same results for powers and efficiency when the load impedances at port 2 and at port 3 satisfy the following relations:

$$R_2 = r_{22}l$$
$$R_3 = r_{33}l, \quad (23)$$

being l a constant.

Table 1. A summary of the parameters values for the approaches that maximize efficiency and power.

Parameter	Maximum efficiency	Maximum power
R_2	$r_{22}\theta$	$r_{22}\theta^2$
R_3	$r_{33}\theta$	$r_{33}\theta^2$
X_{c2}	$-\chi_{13}r_{22}\chi_{23}/\chi_{12}$	=
X_{c3}	$-\chi_{12}r_{33}\chi_{23}/\chi_{13}$	=
R_{in}	$r_{11}\theta$	$2r_{11}\theta^2/(1+\theta^2)$
X_{in}	0	0
p_{in}	$4/\theta$	$2(1+\theta^2)/(\theta^2)$
P_2	$4\chi_{12}^2/(\theta(\theta+1)^2)$	χ_{12}^2/θ^2
P_3	$4\chi_{13}^2/(\theta(\theta+1)^2)$	χ_{13}^2/θ^2
η_{12}	$\chi_{12}^2/(1+\theta)^2$	$\chi_{12}^2/(2(1+\theta^2))$
η_{13}	$\chi_{13}^2/(1+\theta)^2$	$\chi_{13}^2/(2(1+\theta^2))$
$\eta_{tot} = \eta_{12} + \eta_{13}$	$(\theta^2-1)/(1+\theta)^2$	$(\theta^2-1)/(2(1+\theta^2))$

The powers have been normalized with respect to $P_o = V_1^2/(8r_{11})$. R_{in} and X_{in} are the real and the imaginary part of the input impedance (Z_{in}) as seen from the generator (i.e. $Z_{in} = R_{in} + jX_{in}$). p_{in} is the normalized input power. The parameters have the following meanings: $\chi_{12} = x_{12}/\sqrt{r_{11}r_{22}}$, $\chi_{13} = x_{13}/\sqrt{r_{11}r_{33}}$, $\chi_{23} = x_{23}/\sqrt{r_{22}r_{33}}$, $\theta = \sqrt{1+\chi_{12}^2+\chi_{13}^2}$.

This result can be demonstrated as follows. Let us write the impedance matrix of the network in the case of coupled receivers and when the compensating reactances expressed in (20) are present:

$$\mathbf{Z_c} = \begin{pmatrix} r_{11} & jx_{12} & jx_{13} \\ jx_{12} & r_{22} - j\dfrac{x_{13}x_{23}}{x_{12}}\dfrac{r_{22}}{r_{33}} & jx_{23} \\ jx_{13} & jx_{23} & r_{33} - j\dfrac{x_{12}x_{23}}{x_{13}}\dfrac{r_{33}}{r_{22}} \end{pmatrix}. \quad (24)$$

From (9) and (24) we can derive that the admittance matrix ($\mathbf{Y_c}$) is given by:

$$\mathbf{Y_c} = (\mathbf{Z_L} + \mathbf{Z_c})^{-1} = \begin{pmatrix} r_{11} & jx_{12} & jx_{13} \\ jx_{12} & r_{22} + R_2 + jX_{c2} & jx_{23} \\ jx_{13} & jx_{23} & r_{33} + R_3 + jX_{c3} \end{pmatrix}^{-1}. \quad (25)$$

From (20), (25), and (23) it can be verified that the following equalities hold:

$$\begin{aligned} y_{c,11} &= y_{11} \\ y_{c,21} &= y_{21} \\ y_{c,31} &= y_{31}. \end{aligned} \quad (26)$$

As a consequence, we can conclude that when the loads at port 2 and at port 3 satisfy (23), the addition of the compensating reactances expressed in (20) allows to retrieve the same powers and efficiency for both the coupled and the uncoupled case.

From (21) and (18) it can be seen that both the load resistances that maximize power and the ones that maximize efficiency satisfy (23), therefore we can assert that, using the optimal loads, power and efficiency are the same with or without coupling k_{23}, provided that reactances (20) are inserted.

According to the above reported considerations, we derived the maximum efficiency solution for the coupled case by introducing the compensating reactances $X''_{c2} = X'_{c2} = X_{c2}$, $X''_{c3} = X'_{c3} = X_{c3}$, this way we have recovered the values reported in Table 1.

C) Discussion of the results

By comparing results reported in the previous part of this section with those reported in [24], where the same problem is solved for a single-receiver system, we note that the introduction of an additional receiver changes the optimal load impedance. In particular, the load impedance is increased. Therefore, when a system with multiple loads is designed, one should take into account that the optimal impedance should be changed when a further receiver is present.

It is also noted that the solution, which delivers maximum power to the load always presents a larger resistance value with respect to the solution, which provides maximum efficiency. It is this range, i.e. $R''_2 > R_2 > R'_2$, $R''_3 > R_3 > R'_3$, which is the most interesting one for the selection of the load impedances.

Table 2. Two cases of three coupled resonators: for case 1 ($k_{23} = 0$), the terminating impedances are purely resistive. For case 2, all the couplings are considered and the terminating impedances become complex.

Parameter	Case 1	Case 2
f_0	6.78 MHz	=
L_1	4.59 μH	=
L_2	4.59 μH	=
L_3	4.59 μH	=
C_1	120 pF	=
C_2	120 pF	=
C_3	120 pF	=
Q_1	270	=
Q_2	270	=
Q_3	270	=
k_{12}	0.15	=
k_{13}	0.10	=
k_{23}	0.00	0.09

Table 3. Results for three coupled resonators: the element values are listed in Table 2 and refer to case 1. We have assumed that a 1 V generator is present at port 1. It has also been assumed that the generator presents an internal impedance of 50 Ω which has been included in r_{11}.

Parameter	Maximum power	Maximum efficiency
R_2	25.22 Ω	4.27 Ω
R_3	25.22 Ω	4.27 Ω
X_{c2}	0	0
X_{c3}	0	0
P_1	5.07 mW	1.67 mW
P_2	1.66 mW	0.82 mW
P_3	0.73 mW	0.37 mW
P_{tot}	2.39 mW	1.19 mW
η_{12}	0.33	0.49
η_{13}	0.15	0.22
η_{tot}	0.48	0.71

Table 4. Results for three coupled resonators: the element values are listed in Table 2 and refer to case 2. We have assumed that a 1 V generator is present at port 1. It has also been assumed that the generator presents an internal impedance of 50 Ω, which has been included in r_{11}. The first two columns illustrate the results calculated for case 2 without the compensating reactances X_{c2} and X_{c3}, while the last two columns illustrate the results calculated by adding the compensating reactances X_{c2} and X_{c3}.

Parameter	Maximum power	Maximum efficiency	Maximum power	Maximum efficiency
R_2	25.22 Ω	4.27 Ω	25.22 Ω	4.27 Ω
R_3	25.22 Ω	4.27 Ω	25.22 Ω	4.27 Ω
X_{c2}	0	0	−11.7318 Ω	−11.7318 Ω
X_{c3}	0	0	−26.3975 Ω	−26.3975 Ω
P_1	5.64 mW	4.1 mW	5.07 mW	1.67 mW
P_2	1.2 mW	0.32 mW	1.66 mW	0.82 mW
P_3	0.93 mW	0.62 mW	0.73 mW	0.37 mW
P_{tot}	2.13 mW	0.94 mW	2.39 mW	1.19 mW
η_{12}	0.22	0.08	0.33	0.49
η_{13}	0.16	0.15	0.15	0.22
η_{tot}	0.38	0.23	0.47	0.71

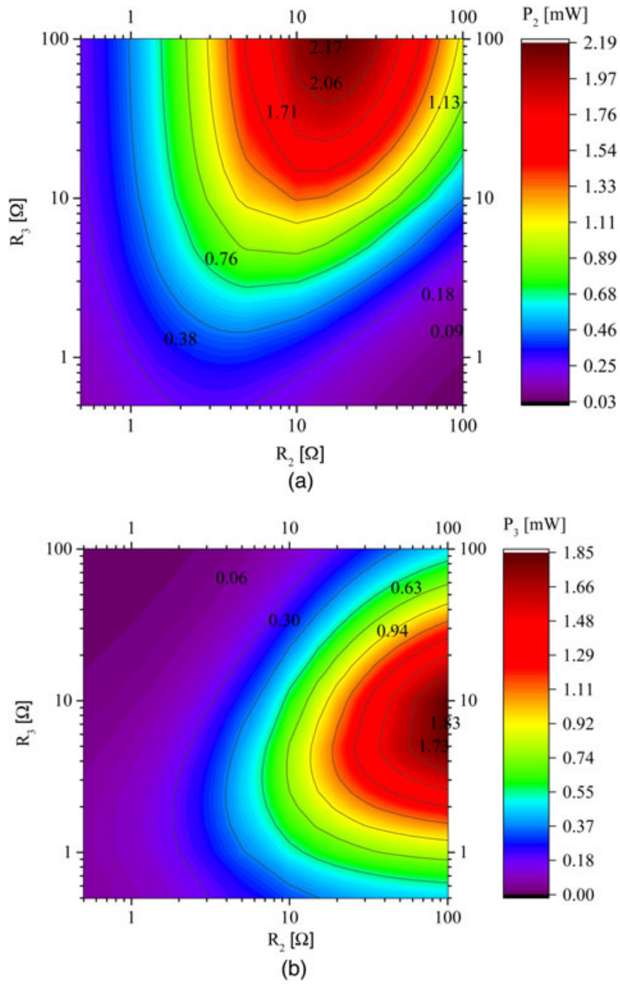

Fig. 3. Power for case 1 as a function of R_2 and R_3. (a) Power on the load at port 2 (i.e. power dissipated on the resistance R_2). (b) Power on the load at port 3 (i.e. power dissipated on the resistance R_3). The results have been obtained by circuit simulations by varying the values of the loads R_2 and R_3. The numeric values on the contour lines indicate the values of the power in mW.

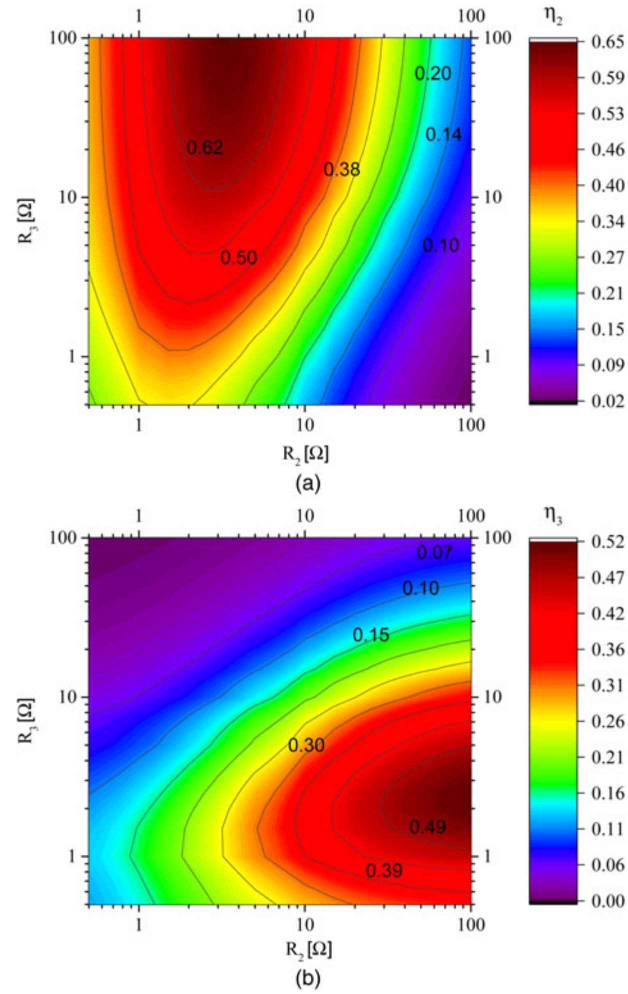

Fig. 4. Efficiency for case 1 as a function of the loads R_2 and R_3. (a) Efficiency with respect to the load at port 2 (i.e. η_{12}). (b) Efficiency with respect to the load at port 3 (i.e. η_{13}). The results have been obtained by circuit simulations by varying the values of the loads R_2 and R_3. The numeric values on the contour lines indicate the values of the efficiency.

IV. NUMERICAL RESULTS

In order to illustrate the proposed approach, two numerical examples have been selected as illustrated in Table 2. Both examples refer to a two receivers WREL and differ only in the mutual coupling between the receivers: in the first example the receivers are not coupled (i.e. $k_{23} = 0$), while in the second example the two receivers are coupled (i.e. $k_{23} \neq 0$). The parameters corresponding to the two examples are gathered in the second column (case 1) and in the third column (case 2) of Table 2.

Using these values, we obtain the following impedance matrix:

$$Z = \begin{pmatrix} 50.724\,\Omega & j29.33\,\Omega & j19.553\,\Omega \\ j29.33\,\Omega & 0.7242\,\Omega & jx_{23} \\ j19.553\,\Omega & jx_{23} & 0.7242\,\Omega \end{pmatrix}, \tag{27}$$

where the parameter x_{23} is equal to zero for case 1, while it is equal to $-17.6\,\Omega$ for case 2. By applying the formulas reported in Table 1 we obtain the results listed in Table 3 for case 1. It can be noticed that the impedances that realize maximum power transfer are greater than the ones that realize maximum efficiency. The results corresponding to the case, where the receivers are coupled (i.e. case 2 of Table 2) are given in Table 4. The first two columns illustrate the results calculated when the compensating reactances are not present, while the last two columns illustrate the results calculated by adding the compensating reactances $X_{c2} = -11.7318\,\Omega$ and $X_{c3} = -26.3975\,\Omega$, which correspond to two series capacitances $C_2 = 2$ nF and $C_3 = 889.26$ pF. From the first two columns of Table 4, it can be noticed that the presence of a coupling between the two receivers affects both the power on the loads and the efficiency. Additionally, it is evident that the efficiency is more sensitive than power to this coupling; in fact, the total efficiency lowers from 0.71 to 0.23. By comparing the last two columns of Table 4 with Table 3, it can be derived that the addition of the compensating reactances allows retrieving the same results calculated for the uncoupled case for both the power on the loads and the efficiency, thus validating the analytical formulas derived in the previous section.

In order to verify results summarized in Table 3 and 4, circuital simulations have been performed by using the

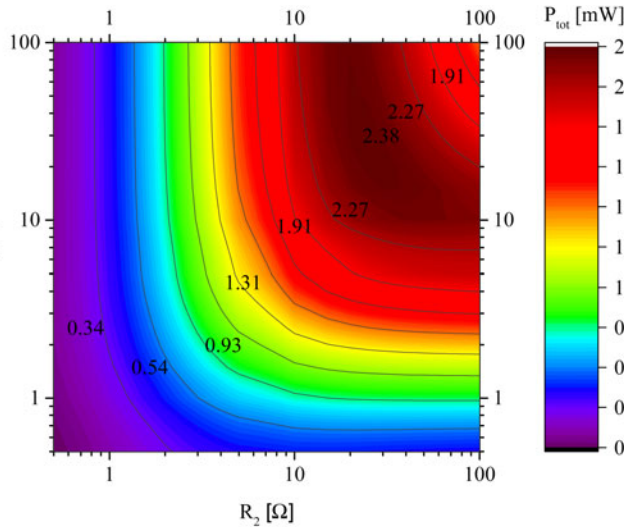

Fig. 5. Total power for case 1 calculated by circuit simulations by varying the values of the loads R_2 and R_3. The numeric values on the contour lines indicate the values of the total power in mW.

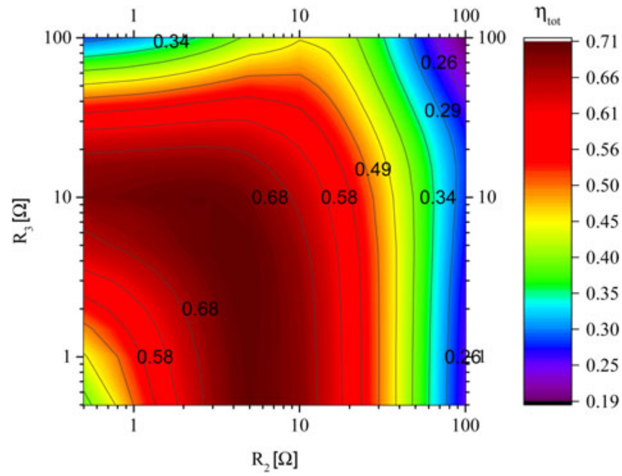

Fig. 8. Efficiency for case 1 calculated by circuit simulations by varying the values of the loads R_2 and R_3. The numeric values on the contour lines indicate the values of the efficiency.

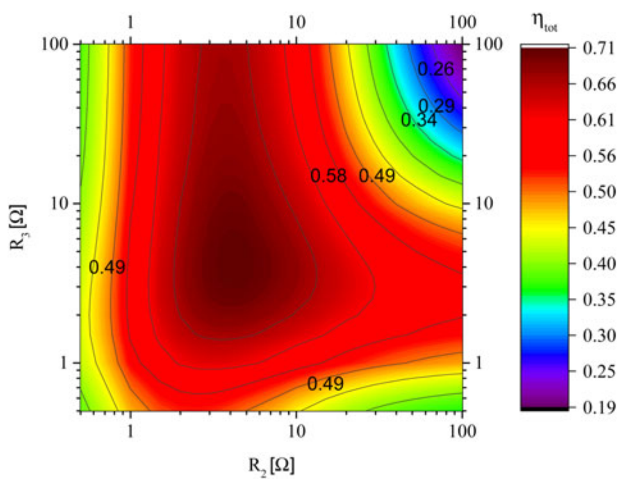

Fig. 6. Efficiency for case 1 calculated by circuit simulations by varying the values of the loads R_2 and R_3. The numeric values on the contour lines indicate the values of the efficiency.

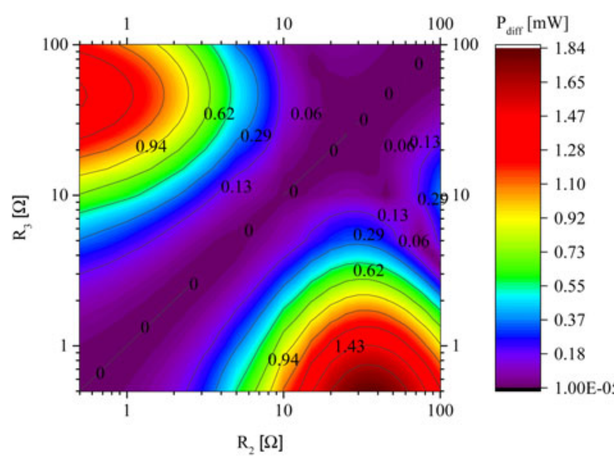

Fig. 9. Difference between the total powers calculated for case 1 and case 2 by varying the values of the loads R_2 and R_3.

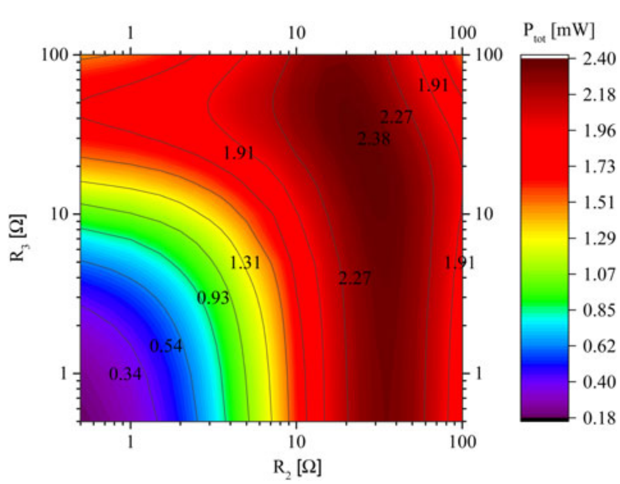

Fig. 7. Total power for case 2 calculated by circuit simulations by varying the values of the loads R_2 and R_3. The numeric values on the contour lines indicate the values of the total power in mW.

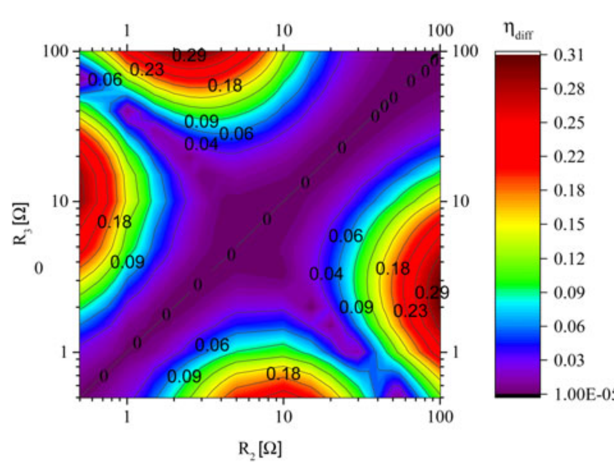

Fig. 10. Difference between the total efficiencies calculated for case 1 and case 2 by varying the values of the loads R_2 and R_3.

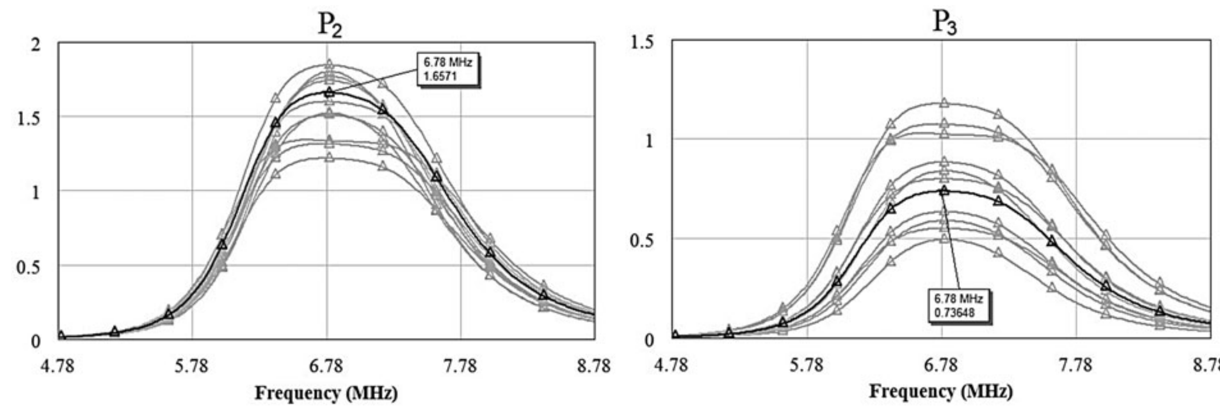

Fig. 11. Sensitivity analysis results calculated by circuit simulations for the powers P_2 and P_3. The values assumed for the loads are the ones corresponding to power maximization (i.e. $R_{L2} = R_{L3} = 25.22\ \Omega$). The black curves are the results calculated by setting the couplings k_{12} and k_{13} at their nominal values (i.e. $k_{12} = 0.15$ and $k_{13} = 0.1$), whereas the gray curves are the results obtained by randomly varying k_{12} in the range (0.12, 0.18) and k_{13} in the range (0.08, 0.12). The powers are in mW.

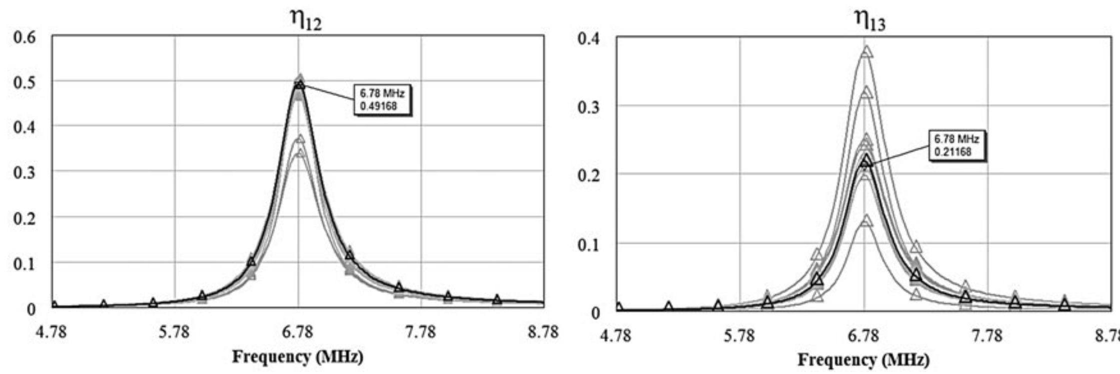

Fig. 12. Sensitivity analysis results calculated by circuit simulations for the efficiencies η_{12} and η_{13}. The values assumed for the loads are the ones corresponding to efficiency maximization (i.e. $R_{L2} = R_{L3} = 4.27\ \Omega$). The black curves are the results calculated by setting the couplings k_{12} and k_{13} at their nominal values (i.e. $k_{12} = 0.15$ and $k_{13} = 0.1$), whereas the gray curves are the results obtained by randomly varying k_{12} in the range (0.12, 0.18) and k_{13} in the range (0.08, 0.12).

commercial tool NI AWR design environment. Corresponding results are given in Figs 3–9. Figures 3–6 refer to case 1; in more detail, Figs 3 and 5 illustrate results obtained for the active power delivered to the loads (i.e. P_2, P_3, and P_{tot}), while Figs 4 and 6 illustrate results obtained for the efficiency. Circuital simulations confirm that the values of the load impedances, which maximize power and efficiency are the ones given in Table 3, thus validating (18) and (21).

Results obtained for case 2 are given in Figs 7 and 8; also in the coupled case circuital simulations are in a perfect agreement with data summarized in Table 4, thus validating theoretical results reported in the previous section. In particular, circuital simulations for case 2 were performed by adding

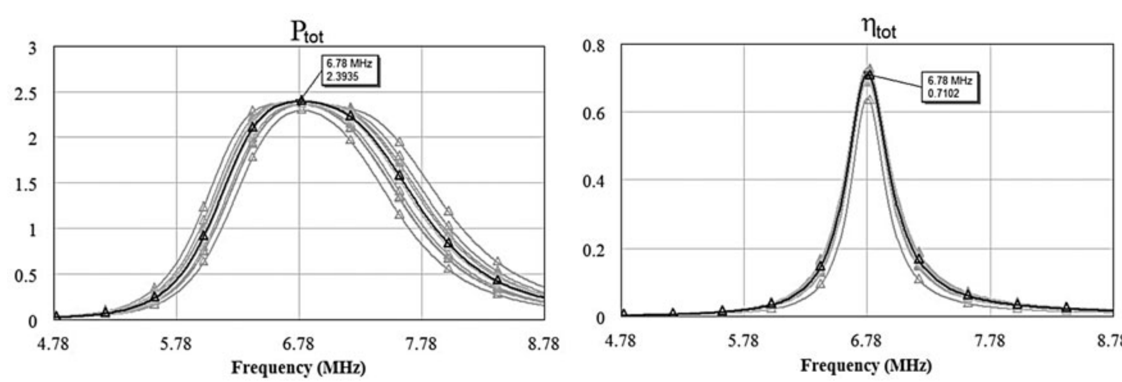

Fig. 13. Sensitivity analysis results calculated by circuit simulations for the total power P_{tot} (on the left) and the total efficiency η_{tot} (on the right). Results referring to P_{tot} have been calculated by using for R_{L2} and R_{L3} the values corresponding to power maximization (i.e. $R_{L2} = R_{L3} = 25.22\ \Omega$), while results referring to η_{tot} have been calculated by using for R_{L2} and R_{L3} the values corresponding to efficiency maximization (i.e. $R_{L2} = R_{L3} = 4.27\ \Omega$). The black curves are the results calculated by setting the couplings k_{12} and k_{13} at their nominal values (i.e. $k_{12} = 0.15$ and $k_{13} = 0.1$), whereas the gray curves are the results obtained by randomly varying k_{12} in the range (0.12, 0.18) and k_{13} in the range (0.08, 0.12). Results reported for the power are in mW.

the compensating reactances X_{c2} and X_{c3} (see Table 4), results obtained this way confirm that these reactances allow to recover in the coupled case the same results obtained for the uncoupled case when the load impedances are the ones for power or efficiency maximization. This result is highlighted in Figs 9 and 10, where the difference between results obtained for case 1 and case 2 are illustrated. In more details, the figures show the quantities P_{diff} and η_{diff}, which are defined as the absolute value of the difference between the values calculated for case 1 and case 2 for the total power and the total efficiency, respectively. It can be seen that this difference is negligible in regions with maximum power or maximum efficiency.

Finally, in order to investigate the dependence of the results reported in Figs 3–8 on possible variations of the couplings among the resonators, a sensitivity analysis has been performed.

More specifically, we assumed for both the k_{12} and the k_{13} parameters a uniform distribution with a 20% standard deviation from their nominal values. Accordingly, circuit simulations have been performed by randomly varying the k_{12} parameter in the range (0.12, 0.18) and the k_{13} parameter in the range (0.08, 0.12).

Results obtained for the case of uncoupled receivers (i.e. $k_{23} = 0$) are summarized in Figs 11–13. As it can be seen, with respect to the values calculated at the nominal values of k_{12} and k_{13}, a large variation has been obtained for the efficiencies η_{12} and η_{13} and the powers P_2 and P_3, while a more robust behavior has been obtained for the total efficiency η_{tot} and the total power P_{tot}.

V. CONCLUSION

The problem of magnetic resonant wireless power transfer between one transmitter and two loads has been considered from a rigorous network viewpoint. Given the network parameters, the problem of maximizing either the power on the loads or the efficiency has been analytically solved. With respect to the use of a circuit simulator to solve the same problem, the main advantages of the proposed analysis is that it provides closed form formulas for the optimal loads, which can be used in order to evaluate the sensitivity of the link performance on the network parameters.

Both cases of coupled and uncoupled receivers have been considered; it has been demonstrated that in the case of coupled receivers the addition of appropriate compensating reactances allows retrieving the same results corresponding to the uncoupled case.

Two numerical simulations have been selected and calculated. The results have indicated that, by introducing an additional receiver, the optimal load impedances (both for maximum efficiency and for maximum power transfer) are changed.

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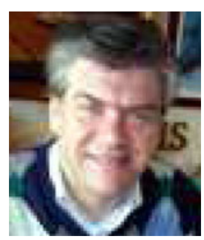

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