Wireless Power Transfer

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Cite this article: Yuan Q, Aoki T (2020). Practical applications of universal approach for calculating maximum transfer efficiency of MIMO-WPT system. *Wireless Power Transfer* **7**, 86–94. https://doi.org/10.1017/wpt.2020.7

Received: 20 April 2019
Revised: 21 December 2019
Accepted: 4 February 2020
First published online: 1 April 2020

Keywords:
Impedance matrix; maximum transfer efficiency; MIMO; optimum load; Rayleigh quotient

Author for correspondence:
Qiaowei Yuan, National Institute of Technology, Sendai College, Ayashi Chuo 4-16-1, Aoba-ku, Sendai, Miyagi, Japan.
E-mail: qwyuan@sendai-nct.ac.jp

Practical applications of universal approach for calculating maximum transfer efficiency of MIMO-WPT system

Qiaowei Yuan and Takumi Aoki

National Institute of Technology, Sendai College, Ayashi Chuo 4-16-1, Aoba-ku, Sendai, Miyagi, Japan

Abstract

In this paper, a concise and universal method to calculate the maximum RF (radio frequency) power transfer efficiency between arbitrary multiple transmitters and multiple receivers wireless power transfer (MIMO-WPT) system is presented. The method is based on maximum Rayleigh quotient which can be deduced either from the multi-port impedance matrix Z or from the multi-port scattering matrix S. Moreover, without any limitation on the transmitting/receiving element's geometry, numbers, operating frequency, coupling method, and so on, the approach is capable to evaluate both the transfer efficiency and the maximum transfer efficiency (MTE) of any type of transmitting and receiving elements, and to obtain the optimum impedances for all transmitting or receiving ports as well. At the end of this paper, the MTEs of some typical MIMO-WPT systems will be calculated to validify the proposed method, and the effectiveness against the receiver's misalignment by using multiple transmitters will be demonstrated.

Introduction

The wireless power transfer (WPT) technology has been developed rapidly because it has broad potential applications to all electronic devices which inevitably need electrical power. The transfer efficiency between the transmitting element (or antenna) and receiving element (or antenna) is a very important factor in evaluating the transfer efficiency of the electromagnetic energy or radio frequency (RF) energy. This efficiency is also called as RF-to-RF efficiency in a WPT system. In the previous researches, most approaches focused on the maximum transfer efficiency (MTE or $\eta_{\max}$) for the single transmitting element and single receiving element (SISO-WPT) [1–5]. In [2, 3], a universal approach to evaluate the MTE for SISO-WPT was deduced perfectly by using S or Z parameters from the ports' conjugated impedance matching condition. The method proposed in [2, 3] can be applied to arbitrary transmitting/receiving elements at the arbitrary operating frequency.

However, delivering power from multiple transmitters to multiple receivers (MIMO-WPT) has many attractive advantages and more useful to practical applications. For example, using more than one transmitter provides a unique capability of focusing energy toward the target receivers, resulting in robustness against many kinds of misalignments mainly caused by mobile receivers. Consequently, more and more researches have focused on MIMO-WPT system [6–14]. In [10], the maximum achievable transfer efficiency of the MIMO system based on Rayleigh quotient by using S-parameters has been described, but the maximum efficiency, optimal input currents and loads have not been explicitly formulated, and there was no practical WPT example to confirm their theory.

Recently, the MTE for single-input multiple-output (SIMO) IPT (Inductive Power Transfer) system operating at a single frequency was presented in [11] based on the deviation deduction mathematical theory, and also was extended to calculate the MTE for MIMO IPT [12]. In [13], the authors presented the mathematical expressions of maximum RF-to-RF efficiency, optimal input currents, and loads for the MIMO system with arbitrary numbers of coils. However, the method in [12, 13] is limited to the case of inductive coils, not a universal one. It is because the approach in [12, 13] used the following Z matrix as

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{R}_{TT} + \boldsymbol{jX}_{TT} & \boldsymbol{jX}_{RT} \\ \boldsymbol{jX}_{TR} & \boldsymbol{R}_{RR} + \boldsymbol{jX}_{RR} \end{bmatrix}, \quad (1)$$

where R_{TT}, X_{TT} are the block matrices representing the self/mutual resistances and self/mutual reactances between the transmitters, respectively. Similarly, R_{RR}, X_{RR} are the block matrices representing the self/mutual resistances, self/mutual reactances between the receivers, respectively. X_{RT} and X_{TR} are the block mutual reactance matrices between the transmitters and receivers.

The real parts of the mutual impedances between the transmitters and receivers are neglected in equation (1). However, generally, the mutual impedance has the real part R_{RT} or R_{TR} as shown in (6). The mutual impedances expressed in (1) are approximately applicable to the coil cases where the imaginary part of the mutual reactance is much larger than the real part.

In [14], the real part of the mutual impedance was included and the expressions for MTE was deducted based on the deviation deduction mathematical theory. Now, in this paper, the approach based on the Rayleigh quotient to obtain the achievable MTE is developed. The Rayleigh quotient in this paper can be formulated not only by impedance matrix Z but also by scattering matrix S. The great advantage over the previous approaches in [11–14] is that the derivation process for MTE is much more concise and clearer. Moreover, the optimal source or load impedances for achieving the MTE are successfully deduced based on the impedance matrix Z.

The important contributions of this paper are the following.

(1) The transfer efficiency of an arbitrary MIMO-WPT system is expressed simply as the Rayleigh quotient which can be constructed not only by the impedance matrix Z but also the scattering matrix S with the very clearly derivation process. Accordingly, the achievable MTE of MIMO-WPT is obtained by solving the maximum eigenvalue of the generalized Rayleigh quotient problem.
(2) The optimal sources and load impedances for achieving the MTE are formulated by using the eigenvector corresponding with the maximum eigenvalue.
(3) The MTEs of several typical examples including SISO-WPT and MIMO-WPT models are calculated to validate the effectiveness of the proposed approach. Particularly, the MTE of SISO is compared with that obtained by our previous method [2, 3].
(4) As a practical application, the misalignment effect on MTEs is investigated for both one transmitter and multiple transmitters to confirm the effectiveness of using multiple transmitters against the receiver's misalignment.

The remainder of this paper is organized as follows: in section "Formulations of MTE for arbitrary MIMO-WPT system", the MIMO WPT system will be introduced and the RF-to-RF MTE will be formulated by using Z-parameter matrix or S-parameter matrix based on the Rayleigh quotient; in section "Validation of the proposed approach", the effectiveness and flexibility of the proposed universal approach will be demonstrated by applying to serval typical MIMO-WPT examples; in section "Against misalignment with using multipole transmitters", to confirm the effectiveness of using multiple transmitters against the receiver's misalignment, the effect on MTE from longitudinal and lateral misalignments of the receiver for $1 \times n$ dipole SISO-WPT, $3 \times I$ dipole MISO-WPT (Multipole Input Single Output), $1 \times d$ loop SISO-WPT, and 3×1 loop MISO-WPT will be investigated by using the proposed method. Finally, the conclusions are presented in section "Conclusion".

Formulations of MTE for arbitrary MIMO-WPT system

By impedance **Z** *matrix*

A typical $M \times N$ MIMO-WPT system with M transmitters and N receivers is shown in Fig. 1.

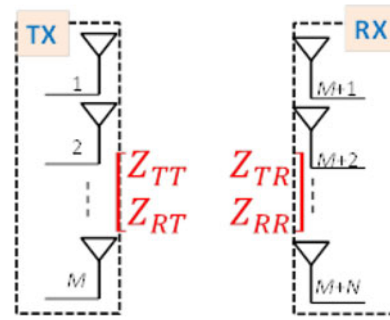

Fig. 1. A typical MIMO system with M transmitters and N receivers.

The voltages and currents at M transmitting ports and N receiving ports have the relationship as expressed by

$$\boldsymbol{V}_T = \boldsymbol{Z}_{TT}\boldsymbol{I}_T + \boldsymbol{Z}_{TR}\boldsymbol{I}_R, \tag{2}$$

$$\boldsymbol{V}_R = \boldsymbol{Z}_{RT}\boldsymbol{I}_T + \boldsymbol{Z}_{RR}\boldsymbol{I}_R. \tag{3}$$

In (2) and (3), V_T, V_R are the M transmitting ports' voltage vector and the N receiving ports' voltage vector, respectively. While I_T, I_R are the M transmitting ports' current vector and the N receiving ports' current vector, respectively. Z_{TT}, Z_{TR}, Z_{RT}, Z_{RR} are the $M \times M$ self-impedance matrix among the M transmitters, $M \times N$ mutual-impedance matrix, $N \times M$ mutual-impedance matrix between the receivers and the transmitters, and $N \times N$ self-impedance matrix among the N receivers, respectively. Certainly, (2) and (3) can be expressed simply in matrix style as

$$\boldsymbol{V} = \boldsymbol{Z}\boldsymbol{I}, \tag{4}$$

where, V and I are the $M+N$ voltage vector and current vector for all transmitting and receiving ports, that is

$$V = \begin{bmatrix} V_T \\ V_R \end{bmatrix}, \ I = \begin{bmatrix} I_T \\ I_R \end{bmatrix}, \tag{5}$$

Z is the $(M+N) \times (M+N)$ impedance matrix which is

$$Z = \begin{bmatrix} Z_{TT} & Z_{TR} \\ Z_{RT} & Z_{RR} \end{bmatrix} = \begin{bmatrix} R_{TT} + jX_{TT} & R_{TR} + jX_{TR} \\ R_{RT} + jX_{RT} & R_{RR} + jX_{RR} \end{bmatrix}. \tag{6}$$

It should be noted that equation (6) is different to equation (1). Equation (6) includes the real parts R_{TR} and R_{RT} in the mutual impedance block matrix without any approximation, but equation (1) did not. Accordingly, the mathematical process for deducing MTE without R_{TR} and R_{RT} in [14] is different from that carried out in [13].

The power transfer efficiency η is the ratio of the output power P_{out} over the input power P_{in}, that is

$$\eta = \frac{P_{out}}{P_{in}}, \tag{7}$$

where, P_{out} and P_{in} represent the total power consumed at all receiving ports and the total transmitting power at all transmitting ports, respectively. P_{out} and P_{in} can be obtained from the port voltage vector and the port current vector which are

$$P_{in} = \frac{1}{2} Re(I_T^H V_T), \tag{8}$$

$$P_{out} = -\frac{1}{2} Re(I_R^H V_R). \tag{9}$$

Instituting equation (8) and equation (9) into equation (7), then the transfer efficiency will be formulated as

$$\eta = -\frac{\begin{bmatrix} I_T \\ I_R \end{bmatrix}^H \begin{bmatrix} 0 & Z_{TR}^* \\ Z_{RT} & 2R_{RR} \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix}}{\begin{bmatrix} I_T \\ I_R \end{bmatrix}^H \begin{bmatrix} 2R_{TT} & Z_{TR} \\ Z_{RT}^* & 0 \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix}} \tag{10}$$

In all equations and formulations in this paper, the superscript $( )^T$ means matrix-transpose, $( )^*$ means complex conjugate, and $( )^H$ means complex conjugate-transpose, respectively.

According to equation (10), the transfer efficiency η is a scalar function of current vectors I_T and I_R, and can be obtained if the load impedance and Z impedance matrix are known. Usually, the load impedance is depended on the receiving consuming devices and can be known previously, while Z impedance matrix also can be known if the operating frequency and the geometries of all transmitting and receiving elements are decided. Z impedance matrix can be obtained with very high accuracy either by using numerical electromagnetic field simulation or by measuring S parameters with a network analyzer.

Obviously, equation (10) can be transformed into the following simple expression,

$$\eta = -\frac{I^H A I}{I^H B I} \tag{11}$$

where,

$$A = \begin{bmatrix} 0 & Z_{TR}^* \\ Z_{RT} & 2R_{RR} \end{bmatrix}, \tag{12}$$

$$B = \begin{bmatrix} 2R_{TT} & Z_{TR} \\ Z_{RT}^* & 0 \end{bmatrix}. \tag{13}$$

The efficiency expressed in equation (11) is the generalized Rayleigh quotient and has the maximum value $\eta_{\max}$ because both matrix A and matrix B are the Hermitian matrices. From the matrix theory of generalized Rayleigh quotient, there exits the maximum value of Rayleigh quotient which is equivalent to the maximum one among the following matrix's generalized eigenvalues,

$$AX = \lambda BX, \tag{14}$$

where λ is the eigenvalue which has $M+N$ values in total. X is the $M+N$ eigenvector, and also totally has $M+N$ vectors related to $M+N$ eigenvalues. Without any approximation and additional hypotheses for Z matrix and for the derivation processes of the above efficiency η and $\eta_{\max}$, consequently, the proposed formulation is the universal one, and can be applied to any type of MIMO-WPT systems which have various combinations according to the transmitters and receivers' geometries, also has no limitation on the operating frequency and coupling approach.

Because the MTE $\eta_{\max}$ is equivalent to the maximum eigenvalue of equation (14), the corresponding eigenvector of the maximum eigenvalue is the optimal current vector to be required at each port for achieving the $\eta_{\max}$. If the corresponding eigenvectors of the maximum eigenvalue are denoted by I_T^E and I_R^E, then the optimum source impedance $(Z_s^{opt})_i$ of i^{th} transmitter port and the load impedance $(Z_l^{opt})_j$ of j^{th} receiver port can be obtained by

$$(Z_s^{opt})_i = \frac{(Z_{TT} I_T^E)_i + (Z_{TR} I_R^E)_i}{(I_T^E)_i}, \tag{15}$$

$$(Z_l^{opt})_j = \frac{(Z_{RR} I_R^E)_j + (Z_{RT} I_T^E)_j}{(I_R^E)_j}. \tag{16}$$

The above formulations mean that the optimum impedances for all transmitters' ports and receivers' ports now can be calculated from the current's eigenvector corresponding to the maximum eigenvalue.

By scattering S matrix

As well known, the scattering S matrix also can express the multi-port circuit's properties of MIMO system as the above Z matrix does. Instead of the voltage vectors and current vectors, the incident wave vectors A_T with M elements at transmitting ports, A_R with N elements at receiving ports, reflected wave vectors B_T with M elements, B_R with N elements have the following relationship as

$$B_T = S_{TT} A_T + S_{TR} A_R, \tag{17}$$

$$B_R = S_{RT} A_T + S_{RR} A_R. \tag{18}$$

where, STT, STR, SRT, SRR are the $M \times M$ block scattering matrix among the M transmitters, $M \times N$ transfer coefficient block matrix from the N receivers to the M transmitters, $N \times M$ transfer coefficient block matrix from the M transmitters to the N receivers, and $N \times N$ block scattering matrix of the N receivers, respectively.

Based on the power efficiency defined by equation (7), following the similar mathematical deduction process as carried out in the above subsection for Z matrix, finally, the following efficiency formulation based on S parameters can be obtained as

$$\eta = -\frac{\begin{bmatrix} A_T \\ A_R \end{bmatrix}^H \begin{bmatrix} S_{TR}^* S_{RT} & S_{TR}^* S_{RR} \\ S_{RR}^* S_{RT} & S_{RR}^* S_{RR} - E_R \end{bmatrix} \begin{bmatrix} A_T \\ A_R \end{bmatrix}}{\begin{bmatrix} A_T \\ A_R \end{bmatrix}^H \begin{bmatrix} S_{TT}^* S_{TT} - E_T & S_{TT}^* S_{TR} \\ S_{RT}^* S_{TT} & S_{RT}^* S_{TR} \end{bmatrix} \begin{bmatrix} A_T \\ A_R \end{bmatrix}}. \tag{19}$$

E_T and E_R in equation (19) are the $M \times M$ unit matrix and $N \times N$ unit matrix, respectively. As a result, the MTE in equation (19) is still the maximum value of the other kind of Rayleigh quotient which is expressed by scattering matrix S. Because the most recent electromagnetic simulation software and the network analyzer preferentially offer the S parameters, equation (19) is much

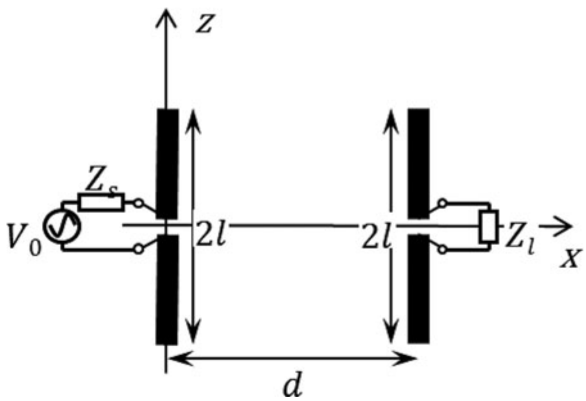

Fig. 2. Dipole pair (SISO system).

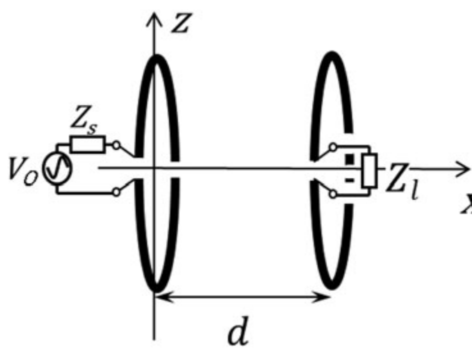

Fig. 3. Loop pair (SISO system).

more a direct way to obtain the efficiency and the maximum efficiency compared with that by using Z matrix. However, to the optimal feeding voltages or optimal load impedance, using the Z matrix is much more simple and direct.

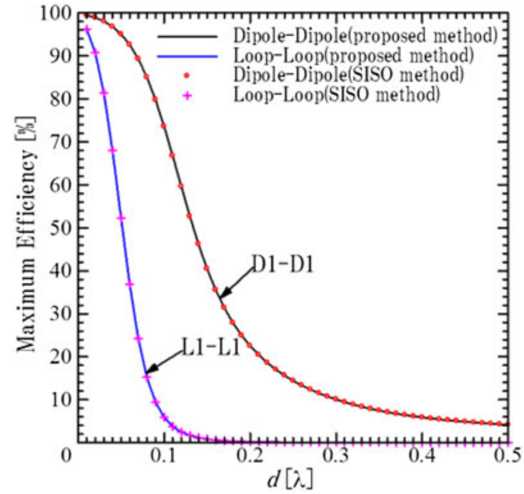

Fig. 4. η_{max} versus *d*.

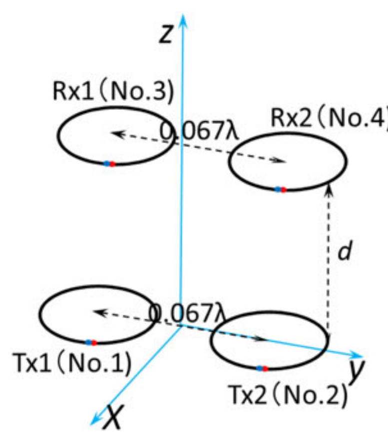

Fig. 5. 2 × 2 MIMO-WPT system.

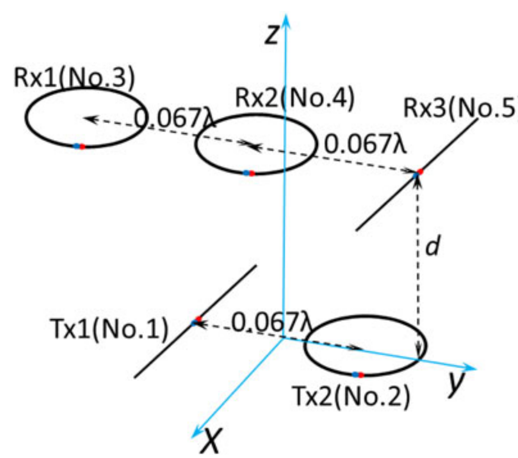

Fig. 6. 2 × 3 MIMO-WPT system.

Validation of the proposed approach

SISO system

The structures in Figs 2 and 3 are two typical SISO systems with one transmitter (T_x) and one receiver (R_x) as used in [3], dipole–dipole pair represents the capacitive coupling, while loop–loop pair represents the inductive coupling. In [3], the results demonstrate the maximum efficiency by using the two-port impedance matrix or scattering matrix which are obtained from the conjugate matching condition at both the transmitting and the receiving ports. In Fig. 2, both T_x and R_x are composed of a dipole with the length of 0.1 wavelengths, while in Fig. 3, both T_x and R_x are composed of a loop with the circumference of 0.1 wavelengths. All wires of the transmitters and the receivers are made of copper with a conductivity of 5.183×10^7 [S/m] and have the same radius of 0.65 mm. The operating frequency is set to be 6.78 MHz.

The MTE versus the distance d between the transmitter and the receiver is shown in Fig. 4. The MTE decreases when the receiver moves away from the transmitter along the x-direction. The MTE of two dipoles is larger than those of loop pair when the distance d is the same. The results obtained by the proposed approach are compared with the MTE obtained by our previous approach [2, 3]. As shown in Fig. 4, it is observed that the two MTEs obtained by two approaches agree with each other perfectly, confirming the maximum eigenvalue of the generalized matrix in equation (14) is the maximum efficiency calculated by our previous approach.

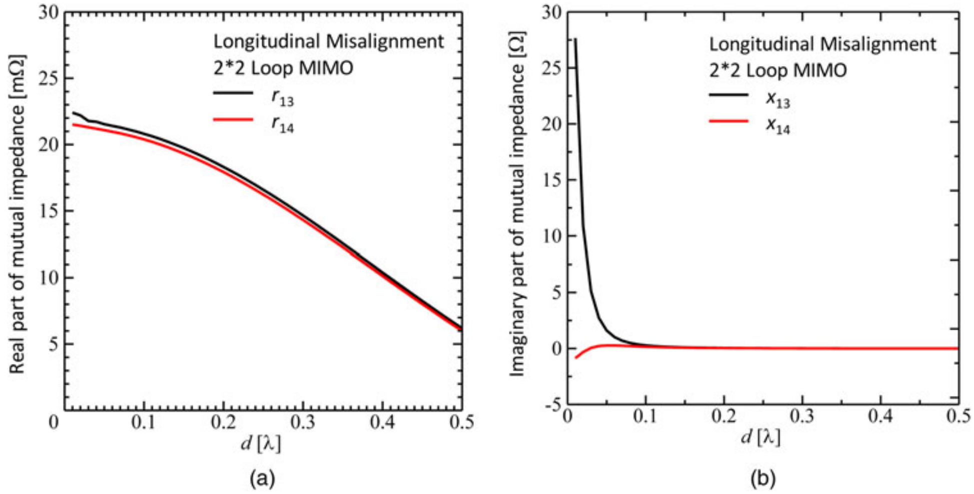

Fig. 7. Mutual impedances of 2 × 2 MIMO-WPT. (a) Real part. (b) Imaginal part.

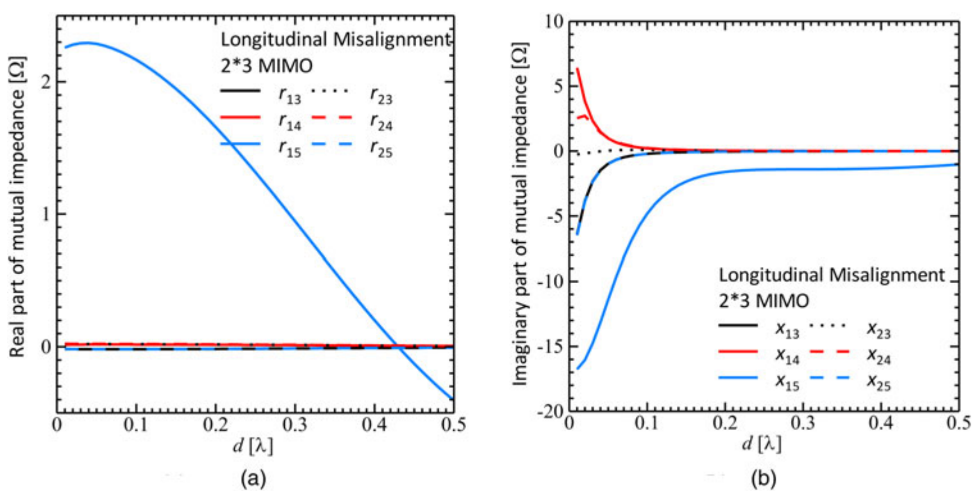

Fig. 8. Mutual impedances of 2 × 3 MIMO-WPT. (a) Real part. (b) Imaginal part.

MIMO system (comparing with the optimal impedance method)

In this sub-section, two MIMO WPT systems, one 2 × 2 MIMO (Fig. 5) and one 2 × 3 MIMO (Fig. 6) are used to confirm the validity of the proposed approach further. In order to confirm the results obtained by the present method, the MTE will be additionally calculated by using the optimal impedance method. In the optimal impedance method, the efficiency is calculated by using the ratio of the total receiving power over the total transmitting power. The total receiving power and the total transmitting power are directly calculated by using electromagnetic field simulation software where the impedances of all transmitter and receiver ports are loaded by the optimal ones which are obtained by using the expressions (15) and (16).

As shown in Fig. 5, 2 × 2 MIMO-WPT system consists of two loop transmitters and two loop receivers. Each loop has the same circumference of 0.1 wavelengths where the operating frequency is selected as 2 MHz. Certainly, the operating frequency can be the arbitrary one which depends on the applications' requirement. All wires of loops are made of copper with a conductivity of 5.183×10^7 [S/m] and have the same radius of 0.5 mm. The two transmitting loops are located at the same plane of $z = 0$, separated by 10 m or 0.67 wavelengths each other along the y-axis. The receiver loops are located at the plane of $z = d$, separated by 10 m or 0.67 wavelengths each other along the y-axis. $Tx1$ and $Rx1$ are coaxial along the z-axis and so are $Tx2$ and $Rx2$. d represents the distance between the center of two transmitters and that of two receivers.

While, the 2 × 3 MIMO-WPT model is shown in Fig. 6 where there are two transmitters and three receivers. One transmitter is the same loop as used in the 2 × 2 MIMO-WPT and the other transmitter is 0.1 wavelength dipole. One receiver is also 0.1 wavelength dipole and the other two receivers are the same loops as the transmitter used. All transmitters are placed on the plane of $z = 0$ and all receivers locate at the plane of $z = d$. d also represents the distance between the plane where the transmitters locate and the plane where the receivers locate as shown in Fig. 6.

The mutual impedances of the 2 × 2 MIMO-WPT and the 2 × 3 MIMO-WPT are shown in Figs 7 and 8, respectively. Because the symmetric structure of the 2 × 2 MIMO-WPT, $z_{13} = z_{24}$, $z_{14} = z_{23}$, only z_{13}, z_{14} are plotted in Fig. 7. Comparing with the dozens mΩ real part of the mutual impedance of the 2 × 2

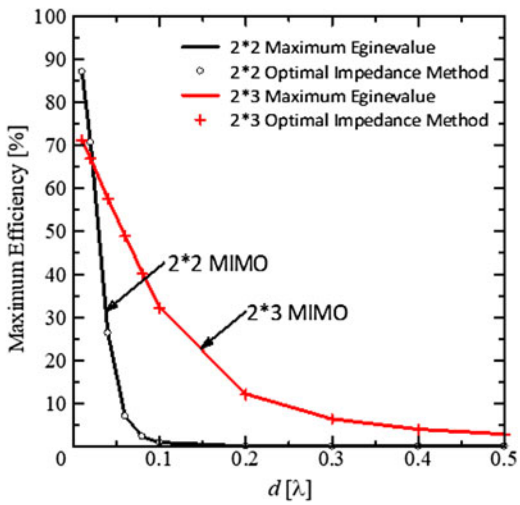

Fig. 9. MTE versus *d* for MIMO systems.

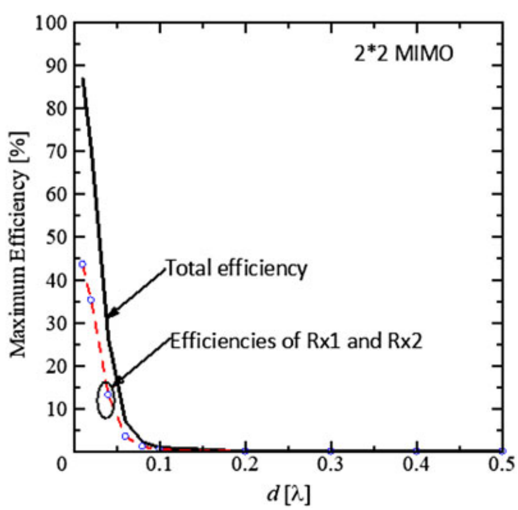

Fig. 10. Each receiver's efficiency of 2 × 2.

MIMO-WPT, the 2 × 3 MIMO-WPT has one larger real part of z_{15} as shown in Fig. 8. z_{15} is the mutual impedance between $Tx1$ (dipole) and $Rx3$(dipole) which represents the mutual impedance of two dipoles and its value changes as the distance between the dipoles changes, and even its real part can be a negative one as shown in [15].

The MTEs versus the distance *d* is shown in Fig. 9 for both 2 × 2 MIMO-WPT and 2 × 3 MIMO-WPT. From Fig. 9, it is found that $\eta_{\max}$ is obtained by the proposed method based on the maximum eigenvalue of equation (14) which is almost the same as that obtained by using the optimal impedance method, again confirming that our proposed approach for obtaining maximum efficiency is still valid even for the above arbitrary MIMO-WPT systems, particularly 2 × 3 MIMO-WPT is one candidate scenario of the arbitrary MIMO-WPT system with both inductive and captive coupling. From Fig. 9, it also can be observed that the efficiency of the 2 × 3 MIMO-WPT drops less than that of the 2 × 2 MIMO-WPT when *d* increases. The reason for that may come from using one more receiving element and using the combination of the inductive coupling of and capacitive coupling in 2 × 3 MIMO-WPT.

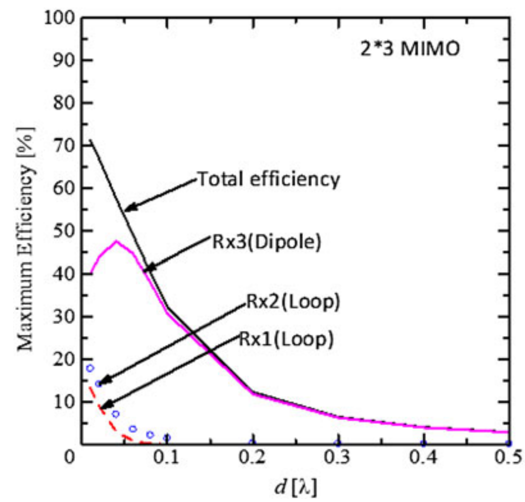

Fig. 11. Each receiver's efficiency of 3 × 2.

Additionally, the individual efficiency of each receiver of 2 × 2 MIMO-WPT system against the distance *d* is illustrated in Fig. 10 when the total efficiency reaches the maximum value. Because the symmetrical location of the transmitters and the receivers, the efficiencies of Rx1 and Rx2 are the same, and each of them is half of the total maximum efficiency.

The individual efficiency of each receiver against the distance *d* for 2 × 3 MIMO-WPT system is illustrated in Fig. 11 when the total efficiency reaches the maximum one. Because the unsymmetrical location of the transmitters and the receivers, the efficiencies of three receivers are different. The dipole receiver accepts the most of the power, nearly attains to 68% of the total receiving power, and other two loop receivers absorb the remaining receiving power with the same receiving efficiency.

Against misalignment with using multipole transmitters

As demonstrated in the above section, the proposed approach for obtaining MTE of the MIMO-WPT system is a universal one, without any limitation in coupling method, transmitter and receiver's geometry, transmitter and receiver's number, operating frequency, and so on. The proposed method enables to be applied to lots of WPT system analysis, evaluation, and design. In this section, the effect of the misalignments between the transmitters and receivers on the maximum efficiency will be investigated for loop coupling and diploe coupling WPT systems, demonstrating the effectiveness of using multiple transmitters to prevent the efficiency decrement caused by the misalignments.

The dipole WPT system is shown in Fig. 12, where all dipoles have the same structure as that used in Fig. 2. The WPT system shown in Fig. 12(a) consists of one transmitting dipole and one receiving dipole (1 × 1 Dipole WPT), while that in Fig. 12(b) consists of three transmitting dipoles and one receiving dipole (3 × 1 Dipole WPT). In Fig. 13, there are the Loop WPT systems. Figure 13(a) is the 1 × 1 Loop WPT and Fig. 13(b) is the 3 × 1 Loop WPT where all loops have the same structures as that used in Fig. 3.

The efficiencies of the dipole WPT and the loop WPT against the distance *d* are shown in Fig. 14, where *d* represents the longitudinal misalignment or the gap between the receiving element and transmitting elements along the *z*-axis. The red dashed

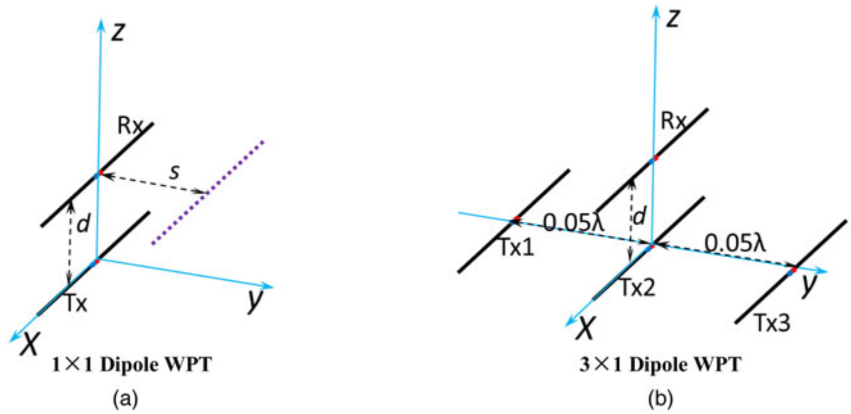

Fig. 12. Dipole MISO-WPT systems. (a) 1 × 1 Dipole WPT. (b) 3 × 1 Dipole WPT.

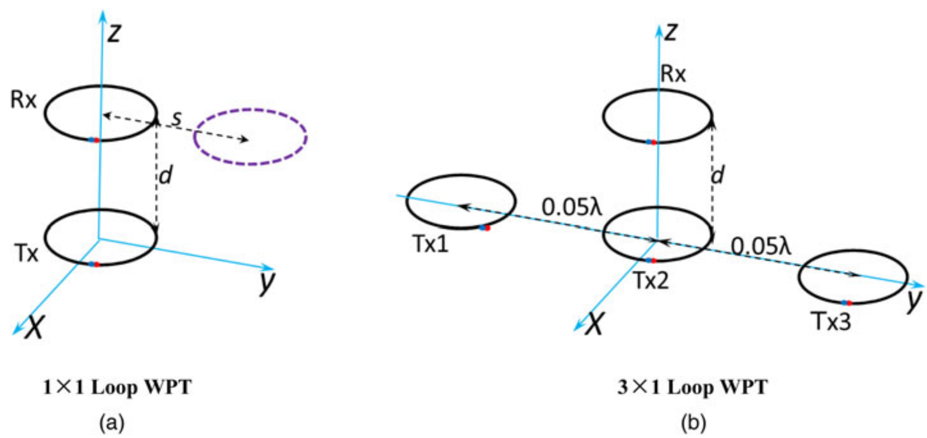

Fig. 13. Loop MISO-WPT systems. (a) 1 × 1 Loop WPT. (b) 3 × 1 Loop WPT.

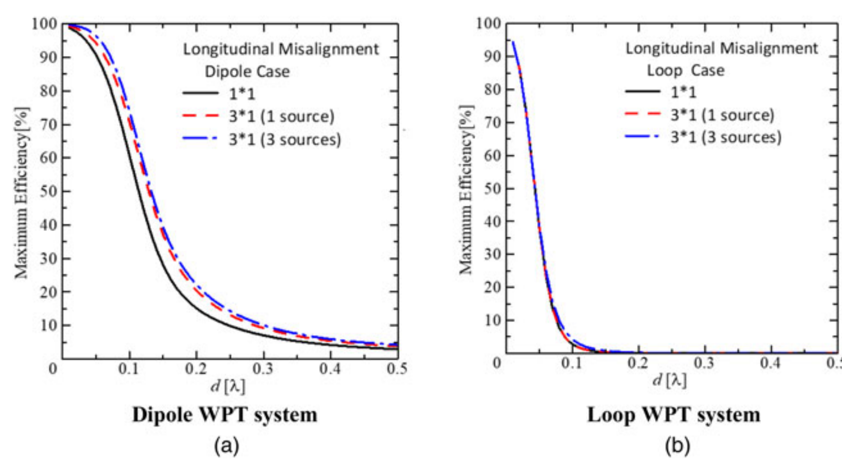

Fig. 14. Efficiency of the MISO system with longitudinal misalignment. (a) Dipole WPT system. (b) Loop WPT system.

lines represent the efficiency when three transmitting elements are used, but only the middle element is fed and the other two elements are used as the parasitic elements without feeding sources. The blue lines represent the efficiency when three transmitting elements are used and all elements have feeding sources. When the receiving element is moved far away from the transmitting elements, the efficiencies in all cases are decreased. There is no significant improvement in the efficiency when three transmitting elements are used for loop WPT system. However, in the dipole WPT case, the efficiency using three transmitting dipoles is a bit higher than those of the other two cases.

The efficiencies of loop WPT and dipole WPT against the distance *s* are shown in Fig. 15, where *s* represents the lateral misalignment of the receiving element along the *y*-axis. Either for the dipole case or for the loop case, using three elements has improved the efficiency greatly, demonstrating that using multiple transmitting elements is an effective way against the lateral misalignment.

From the above simulation results for the two fundamental misalignment scenarios, it is found that using multiple transmitting elements can prevent the efficiency dropping caused by the misalignment because more transmitting elements' excitations

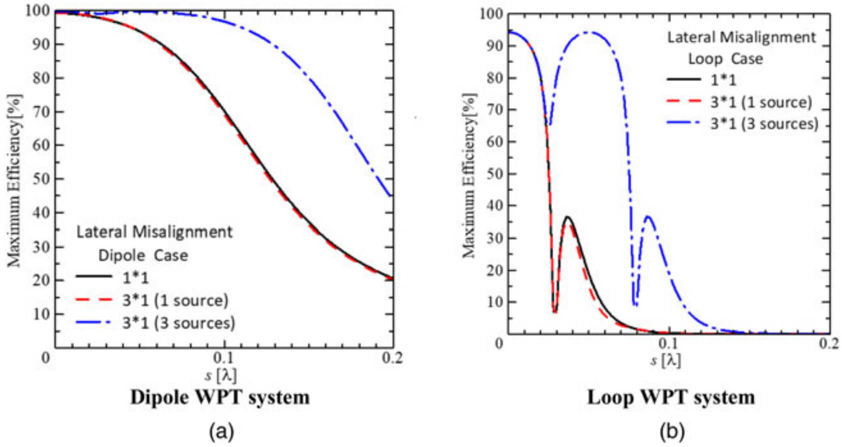

Fig. 15. Efficiency of the MISO system with lateral misalignment. (a) Dipole WPT system. (b) Loop WPT system.

can be adjusted effectively to concentrate the power to the receiving element. But at the same time, it should be noted that the efficiency improvement depends on the misalignment direction and the transmitter or receiver's structures as well, meaning that there is an optimal WPT system structure against the misalignment.

Conclusion

In this paper, the efficiency of MIMO-WPT system has been expressed by the Rayleigh quotient either from multi-port impedance Z matrix or from scattering S matrix. Consequently, a very concise and universal approach to calculate the power transfer efficiency and also its maximum one has been presented successfully. The effectiveness and flexibility of the new approach have been demonstrated by calculating the MTEs of eight MIMO-WPT systems including four SISOs, one 2×2 MIMO, one 2×3 MIMO, and two 3×1 MISO WPT systems. Moreover, it has been demonstrated that using multiple transmitters is an effective way against the receiver's misalignment by investigating the misalignment on the MTE of MISO-WPT by using the proposed method.

Acknowledgements. This work was partly supported by JSPS KAKENHI Grant Number 18K04116.

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Qiaowei Yuan received BE, ME, and PhD degrees from Xidian University, Xi'an, China, in 1986, 1989, and 1997, respectively. From 1990 to 1991, she was a special research student at Tohoku University, Sendai, Japan. From 1992 to 1995, she worked in Sendai Research and Development Laboratory, Matsushita Communication Company, Ltd., engaged in the research and design of compact antennas for second-generation mobile phones. From 1997 to 2002, she was a researcher in Sendai Research and Development Center, Oi Electric Company, Ltd., involved in the researching and designing of small antennas for pager communications and the parabolic antenna for 26.5 GHz fixed wireless access (FWA) communication. From 2002 to 2007, she was a researcher with Intelligent Cosmos Research Institute, Sendai, Japan, involved in the research and development of adaptive array antennas and RF circuits for mobile communications. From 2007 to 2008, she was an Associate Professor of Tokyo University of Agriculture and Technology. She is currently a Professor of the National Institute of Technology, Sendai College. Dr. Yuan received the Best Paper Award and Zenichi Kiyasu Award in 2009 from the Institute of Electronics, Information and Communication Engineers (IEICE) of Japan. She served as the secretary of IEICE Technical Committee on Wireless Power transfer of Japan from 2013 to 2015. Also, she was a visiting scholar at ElectroScience Laboratory of Ohio State University in the United States from April 2015 to September 2015.

Takumi Aoki is a first-grade student of an advanced course in the National Institute of Technology, Sendai College, Japan. He began a research study on wireless power transfer technology 3 years ago and now he has enrolled for a 3-month internship in Menicon Innovation Center in Geneva.